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ELEMENTARY COURSE IN PERSPECTIVE

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Fig. 12, Scale $1' = 30'$ to $1'' = 30'$
Fig. 13, Scale $1' = 8'$ to $1'' = 8'$

ELEMENTARY COURSE IN PERSPECTIVE

By

SHERMAN M. TURRILL

Civil Engineer, Assoc. Am. Soc. C. E.

WITH 16 *ILLUSTRATIONS AND FOLDING PLATES*

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PREFACE

In the following pages the author has endeavored to illustrate the mechanical application of the principles of Descriptive Geometry to the making of a perspective drawing. Two methods are used: the "Method by the Use of the Plan," where the orthographic projections of rays are used, and the "Method by Scale." In the former, all details should be drawn with instruments, especially if there is much mechanical labor entailed. On the other hand, the "Method by Scale" should serve the purpose, and is, in fact, the quicker process, where the accurate location of the leading or important points only is desired. The first method is discussed mainly for the assistance of the mechanical draftsman, whereas the second is intended for the artist and free-hand draftsman.

In writing this treatise the author has assumed that the student is more or less familiar with Descriptive Geometry. The subjects taken have been selected with a view toward illustrating important principles and for their general interest.

In addition will be found a large number of problems covering these principles as applied to the illustrated subject, in a variety of positions.

SHERMAN M. TURRILL.

CHICAGO, Ill.,
January, 1910.

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PART I

PERSPECTIVE BY ORTHOGRAPHIC PROJECTION

CHAPTER I

PERSPECTIVE

1. THE perspective of any object is its projection on a vertical plane when the point of sight is at a finite distance from the object and the plane of projection, which is commonly called the picture plane, as *abcd*, Fig. 1.

Point of sight or station is the point from which the object is viewed and is the apex of a cone from which all rays from different points of the object have their origin, as *S*, Fig. 1. Hence the name conical projection, which is sometimes given as the definition of perspective.

Picture plane is the vertical plane upon which the perspective is drawn and is usually between the object and the point of sight, as P. P., Fig. 1. If it is desired to have the perspective larger than the object the picture plane is placed behind the object and the rays of light pass from the point of sight through the object to the picture plane behind.

The horizon is the line of intersection of the picture plane with a horizontal plane through the point of sight, as shown in Fig. 1. These two planes are always perpendicular to each other.

The center of picture is the orthographic projection of the station on the picture plane, as s^v, Fig. 1.

The vanishing point of any line or system of lines is found by passing a line through the station parallel to the line or system of lines, and the point where this line intersects or pierces the picture plane is the vanishing point of the line or system of lines. In other words it is the perspective of infinity, the point where the parallel lines of any system meet. If the lines are horizontal their vanishing point is in the horizon, as V. P., Fig. 1. If not horizontal the vanishing point is above or below the horizon as $(V. P.)^1$ in P. P., Fig. 1.

By indefinite perspective of a line is meant the perspective of a line which extends from the picture plane to infinity. c^v V. P., Fig. 1, is the perspective of such a line, c^v being the perspective of the point of the line *CB* in the picture plane, and V. P. being the perspective of that point of the line *CB* which is at infinity, Fig. 1.

The definite perspective of a line is the perspective of a definite portion of an indefinite perspective and is any portion of the indefinite perspective. Thus the perspective bc is a portion of the indefinite perspective c^v V. P., and hence the definite perspective of $B\,C$, Fig. 1.

Parallel perspective of any object is the perspective of that object when at least one of the faces of a parallelopiped enclosing the object is in or parallel to the picture plane as shown in Fig. 1. pictorially and in Figs. 2 and 3 orthographically.

NOTATION

II. Station or Point of Sight is represented by . . . . S.
Picture Plane is represented by . . . . P. P.
Horizon is represented by . . . . Hor.
Horizontal Plane is represented by . . . . H.
Vanishing Point is represented by . . . . V. P.

Points themselves are represented by capital letters, and their projections by small letters with the power h. or v. designating horizontal or vertical projections respectively.

Lines are represented by two points.

Planes are named by a capital letter, and represented by their traces, these traces being lettered with the prefix H. or V. placed before the capital letter of the plane to designate horizontal and vertical trace of the plane.

Where a line lies in an oblique plane, it will intersect the horizontal and vertical planes of projection if produced, and where it intersects these planes of projection is lettered h. i. and v. i. respectively.

Center of picture is the vertical projection of the station.

Chapter II

PARALLEL PERSPECTIVE

III. In the pictorial illustration, Fig. 1, let *ABCDE* represent a pyramid in space, one of the faces $c^h d^h$ *C D* enclosing it being parallel to the picture plane. The orthographic projection of this object in the horizontal plane H. is $a^h b^h c^h$ etc. Its vertical projection in the picture plane P. P. is $a^v c^v d^v$. The horizontal projection of *S*, the station, is s^h, and of the rays *S B*, *S D*, etc., is $s^h b^h$, $s^h d^h$, etc., which intersect *XX* in b^x, d^x, etc. From these points erect perpendiculars to H. and they lie in the picture plane P. P. These last lines, if produced far enough, will intersect their corresponding rays in *b*, *d*, etc., which are the perspective of the points *B*, *D*, etc. If these points in the perspective are joined by lines in the same sequence as the original object we will have the perspective *abcd* of the object *ABCDE*.

The method by vanishing points is the one used in practice and will now be described by the use of Fig. 1. The steps are the same as above up to and including the perpendiculars in P. P. In order to obtain the vanishing point (V. P.) pass through the station S, a straight line parallel to CB and DE. The horizontal projection of this line through S is $s^h s^x$ parallel to $c^h b^h$.* From s^x erect a perpendicular to H. intersecting the parallel line from S in s^v. This is where the parallel line through S intersects the picture plane P. P., and is the vanishing point of the system of lines parallel to CB, and is the perspective of infinity where these parallel lines meet. The line $C B$, if produced, intersects the picture plane in c^v, and the indefinite perspective of the line $C B$ is c^v V. P., and the definite perspective of that portion of the line from C to B is cb, found by dropping perpendiculars in P. P. from b^x and c^x until they intersect the indefinite perspective c^v V. P. in b and c. The same method of reasoning is followed for the line DE and a line through A parallel to $D E$, and thus a and d are found. To complete the perspective, lines are drawn from c to d, d to a, c to a, and b to a, and we have the perspective $abcd$ of the object $A B C D$. As a check on the work cd should be parallel to $c^v d^v$ and XX since $C D$ is parallel to H. and P. P., and therefore a line through S parallel to $C D$ meets P. P. at infinity, and therefore the perspective of $C\,D$ is parallel to itself. As another check we can find the vanishing point of $A\,C$. It is found by passing a line through S parallel to $A\,C$, and where it intersects P. P. is the vanishing point of $A\,C$. To find this pass through the horizontal and vertical projections of S, i.e., s^h and s^v, straight lines $s^h o^x$, s^v (V. P.)^1 parallel to the corresponding projections of $A\,C$, i.e., $a^h c^h$, $a^v c^v$.

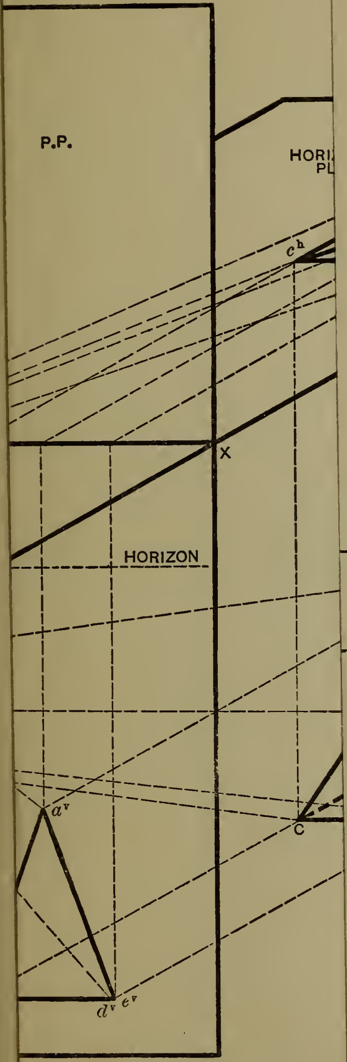
---

* If two lines are parallel in space their corresponding projections are parallel.

From o^x in $X\,X$ erect a perpendicular to H. in P. P. and it intersects the line through s^v in (V. P.)^1, which is the vanishing point of $A\,C$. Hence when c is determined, draw the line c (V. P.)^1, and it should coincide with *ca*, and likewise we can find the vanishing points of $A\,D$ and $A\,E$.

IV. The orthographic projection of the perspective *abcd* is shown in Fig. 2. The same points bear the same letters as in Fig. 1, so that the above description for the method of vanishing points will describe this figure, only bearing in mind that the horizontal and vertical planes of Fig. 1 are so swung that the part of H. in front of $X\,X$ and that part of P. P. below XX coincide and are below $X\,X$ while that part of H. back of $X\,X$ and that part of P. P. above $X\,X$ also coincide, and are above $X\,X$ as shown in Fig. 2.

V. Figure 3 gives the orthographic projection of a more complicated problem of parallel perspective than was found in Fig. 2. The $X X$ axis or horizontal projection of the picture plane and the plan of the building $a^h c^h j^h m^h$ are located so that one end of the building is parallel to the picture plane. The horizontal projection of the station point s^h is so chosen at random that a good perspective is obtained, and the horizontal projection of the rays $s^h a^h$, $s^h c^h$, $s^h d^h$, etc., are drawn intersecting the $X X$ axis in a^x, c^x, d^x, etc., respectively. The edges of the building or lines $A M, P L$, $C J$, etc., are produced until they intersect the picture plane and the vertical intersection of these lines are shown in a^v, p^v, c^v, etc., respectively. The points are connected as shown, and we get the end of the building in its true shape and size. The vanishing point, of the system of parallel lines $A M$, $P L$, $C J$, etc., is found by passing a line through S parallel to them and where it intersects the picture plane is the vanishing point. We therefore draw a line through s^h, parallel to $a^h m^h$, $p^h l^h$, etc., intersecting the $X X$ axis in s^x, and the perpendicular from s^x to the intersection with the horizon gives V. P., since the system of lines is horizontal. The indefinite perspective of the above system is then found or shown by a^v V. P., p^v V. P., c^v V. P., etc. The definite


P.P.
HORI
PL
c h
X
HORIZON
a v
C
d v e v

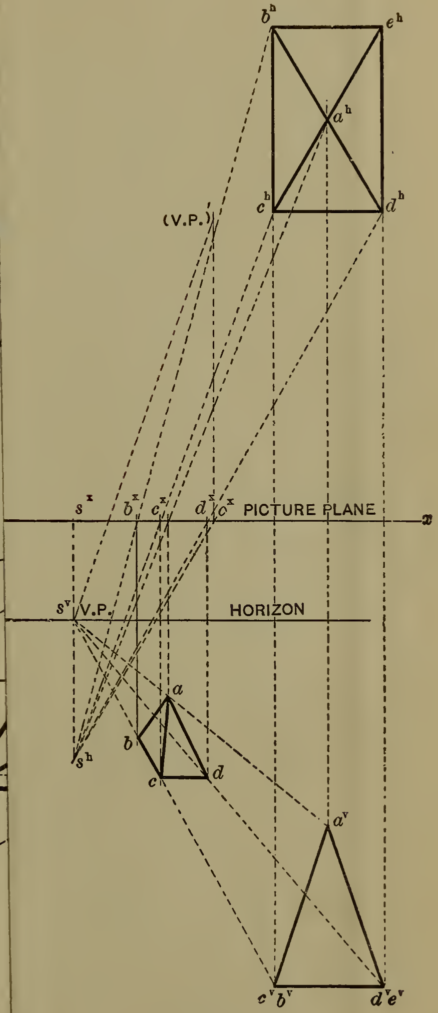

b h
e h
a h
c h
d h
(V.P.)
s x
b x
c x
d x
c x
PICTURE PLANE
x
s v
V.P.
HORIZON
a
b
c
d
s h
a v
c v b v
d v e v

Fig 2

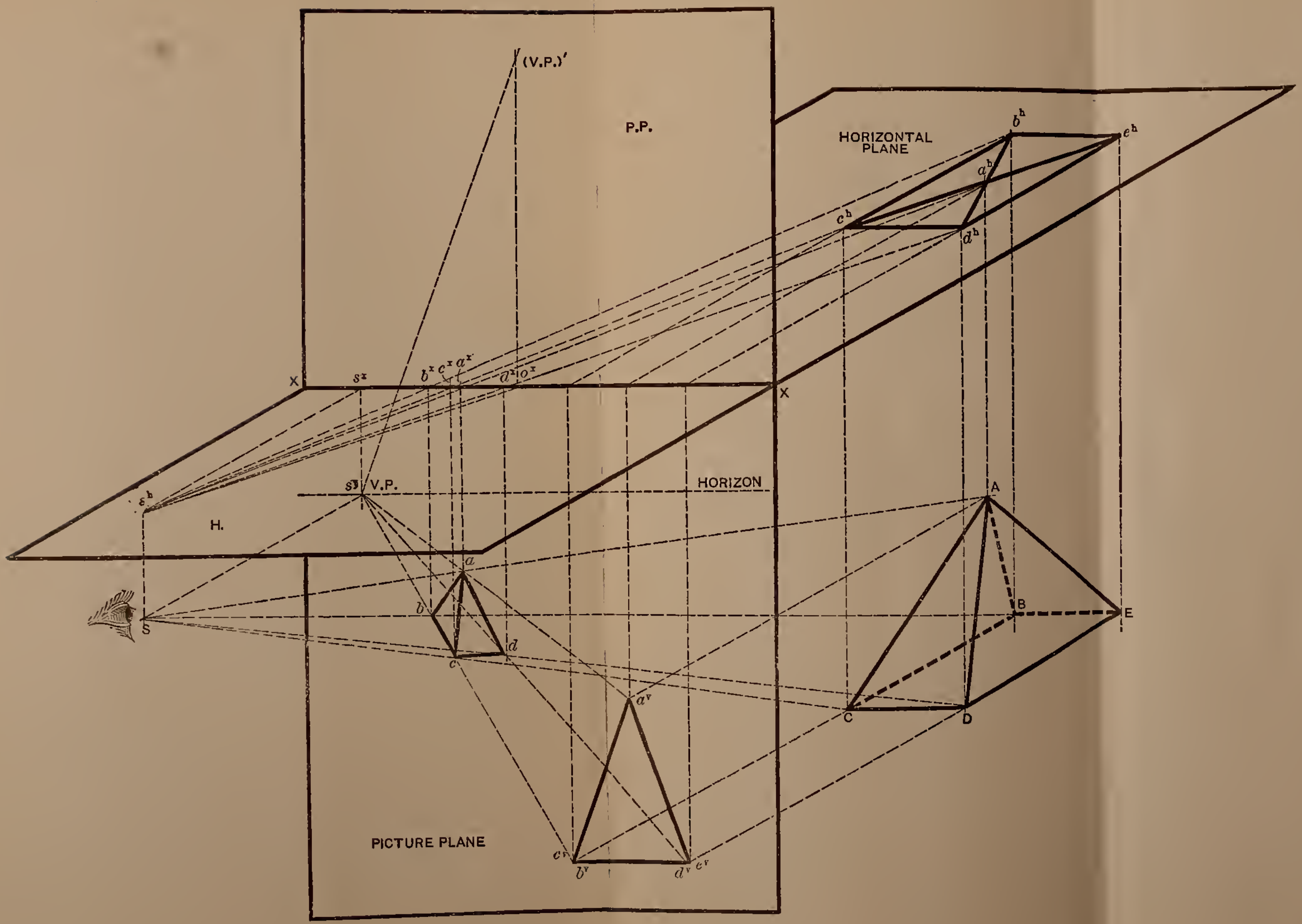

(V.P.)'
P.P.
HORIZONTAL PLANE
b^h
e^h
a^h
c^h
d^h
X
s^x
b^x c^x a^x
d^x o^x
X
s^h
s^v V.P.
HORIZON
H.
A
B
E
C
D
a
b
c
d
S
a^v
c^v
b^v
d^v e^v
PICTURE PLANE

Fig. 1.

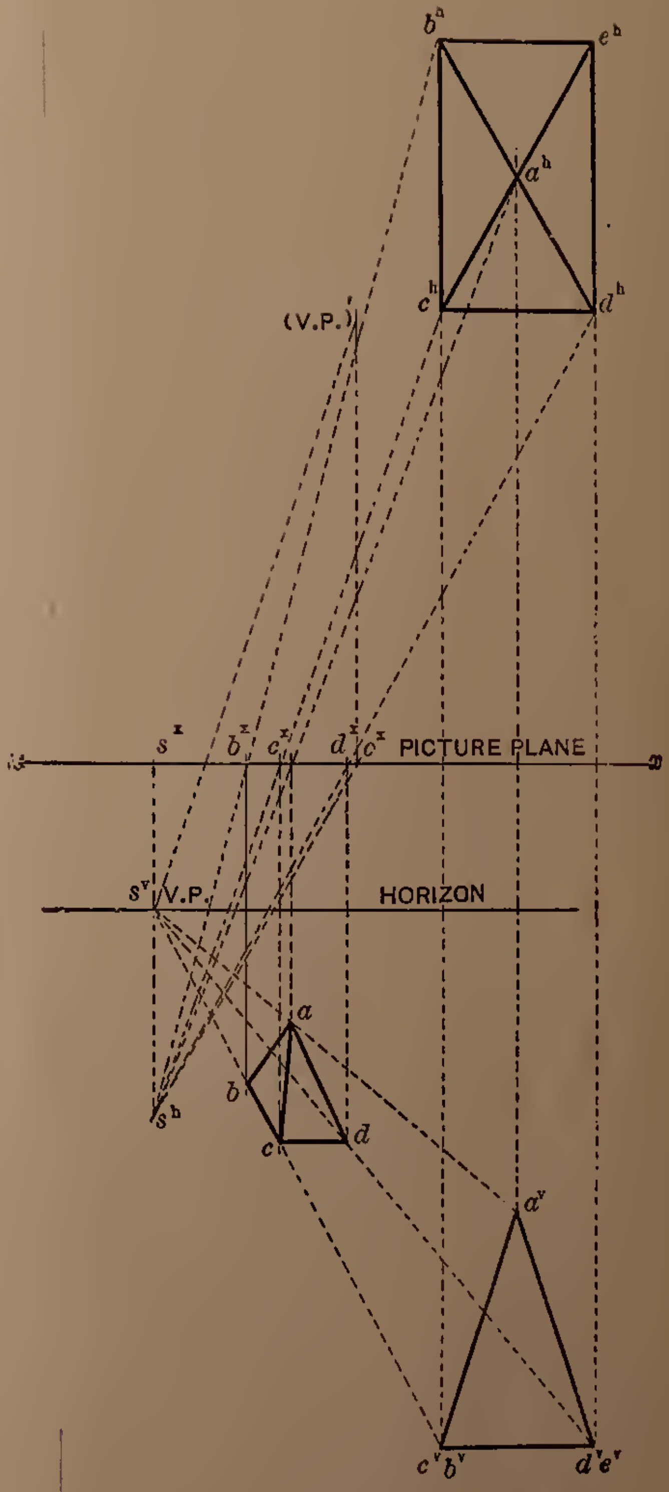

b^h
e^h
a^h
c^h
d^h
(V.P.)
x
s^x
b^x
c^x
d^x c^x
PICTURE PLANE
x
s^v V.P.
HORIZON
a
b
c
d
s^h
a^v
c^v b^v
d^v e^v

Fig. 2

perspectives of these lines are then found by dropping perpendiculars from a^x, (m^x not shown), p^x, l^x, c^x, j^x, etc., until they intersect their respective indefinite perspectives, and that portion included between the respective points is the definite perspective a (m not shown), $p\,l$, $c\,j$, etc. To get the perspective of R draw the horizontal projection of a ray from r^h, and where this ray intersects the $X\,X$ axis in r^x erect a perpendicular, and where it intersects the indefinite perspective r^v V. P. will be the perspective of R represented by r. Join cr and we have the perspective cr of $C\,R$. The perspectives of $O\,A$, $J\,K$, etc., are found in the same way. The perspectives of $P\,R$, $P\,O$, etc., are found by first obtaining the perspective p and then join r and p, also o and p, etc. The perspectives or, cc_1, aa_1, etc., are all perpendicular or parallel to the horizon, for horizontal or perpendicular lines parallel to the picture plane bear the same relation to the horizon in perspective.

The perspective of the wing is found as follows. Produce a horizontal line through F and I until it intersects the picture plane and is represented by f^v. The indefinite perspective of FI is f^v (V. P.), and the definite perspective fi is found as described for other lines. The perspective fd is a line parallel to the horizon, and the perspective *d* is found by drawing a ray $d^h s^h$, and where it intersects *X X* in d^x drop a perpendicular, and where it crosses the horizontal line through *f* gives *d*. By following these principles for each point the entire perspective is constructed. The perspectives *op*, *pr*, *rc*, etc., could be obtained by finding the vanishing points of these lines, and as an illustration of this method we will take the line *R P*. $r^h p^h$; $r^v p^v$ are the horizontal and vertical projections of *R P*. Therefore to find the vanishing point of this line pass through the station a line parallel to *R P*. The projections of this line through the projections of the station are $s^h t^h$, $s^v t^v$, and will be parallel to the projections $r^h p^h$; $r^v p^v$, respectively. Where this line *S T* intersects the picture plane will be the vanishing point of the line *R P*, and it is represented by $(\text{V. P.})^1$, and the perspective of *R P* lies on the line *r* $(\text{V. P.})^1$ and coincides with the line *rp* already drawn. In like manner the perspective of any line in the drawing can be found.

PROBLEMS IN PARALLEL PERSPECTIVE

VI. 1. Find the parallel perspective of Fig. 3 when the end of the building is in the picture plane.

2. Find the perspective of Fig. 3 when the building is turned through 90° and *FI* is parallel

to the picture plane, and find the vanishing point of at least one system of oblique lines.

3. Find the parallel perspective of Fig. 3 when *FI* is in the picture plane, and find the vanishing point of at least one system of oblique lines.

4. Find the perspective of Fig. 3 when the station is moved as far to the left as it is to the right of the center of the picture.

5. Find the parallel perspective of Fig. 3 when the wing is removed and the end of the roof has no triangular sloping surface, and find the vanishing point of at least one system of oblique lines.

6. Find the parallel perspective of Fig. 4 when *A B* is parallel to the picture plane.

7. Find the parallel perspective of Fig. 4 when *A B* is in the picture plane, and find the vanishing point of at least one system of oblique lines.

8. Find the parallel perspective of Fig. 4 when *B C* is parallel to the picture plane, and find the vanishing point of at least one system of oblique lines.

9. Find the perspective of Fig. 4 when *B C* is in the picture plane.

10. Find the perspective of Fig. 4 when *A B* is parallel to the picture plane and the parallelopiped *EI* rests on the right side of the cube or on the edge *B C*.

11. Find the perspective of Fig. 4 when *A B*

is parallel to the picture plane and the block *EI* rests on the edge *A B,* and the entire object is behind the picture plane; and find the vanishing point of at least one system of oblique lines.

12. Find the perspective of Fig. 4 when an edge of the cube is parallel to the picture plane, but the object is in front of the picture plane instead of behind it.

13. Find the perspective of Fig. 3 when the object is in front of the picture plane, and find the vanishing point of at least one system of oblique lines.

14. Find the perspective of a right parallelopiped when one face is parallel to the picture plane and one end is the base of the object.

15. Find the perspective of a right parallelopiped when one face is parallel to the picture plane and one face as the base of the object, the ends being both perpendicular to the base and the picture plane.

16. Find the perspective of a right parallelopiped when the end is parallel to the picture plane.

17. Find the perspective of a right parallelopiped when one face is in the picture plane.

18. Find the perspective of a right octagonal prism when one edge is parallel to the picture plane and the octagon is the base.

19. Find the perspective of a right octagonal

prism when one face is parallel to the picture plane and the octagon is the base.

20. Find the perspective of a right octagonal prism when one edge is in the picture plane and the octagon is the base.

21. Find the perspective of a right octagonal prism when one edge is parallel to the picture plane and one of the rectangular faces is the base, and find the vanishing point of at least one system of oblique lines.

22. Find the perspective of a right octagonal prism when one face is parallel to the picture plane and an edge is the base.

23. Find the perspective of a right octagonal prism when one face is in the picture plane and an edge is the base, and find the vanishing point of at least one system of oblique lines.

24. Find the perspective of a right octagonal prism when one edge is in the picture plane and one of the rectangular faces is the base.

25. Find the perspective of a parallelopiped block with a mortise in one face and one end of the solid is halved.

26. Find the parallel perspective of a parallelopiped block when one end has a tenon and the other square.

27. Find the perspective of a right square pyramid.

28. Find the perspective of an oblique square pyramid and find the vanishing point of at least one system of oblique lines.

29. Find the perspective of a tee made up of two parallelopipeds of the same size and with square cross-sections.

30. Find the perspective of the frustum of an oblique square pyramid, and find the vanishing point of at least one system of oblique lines.

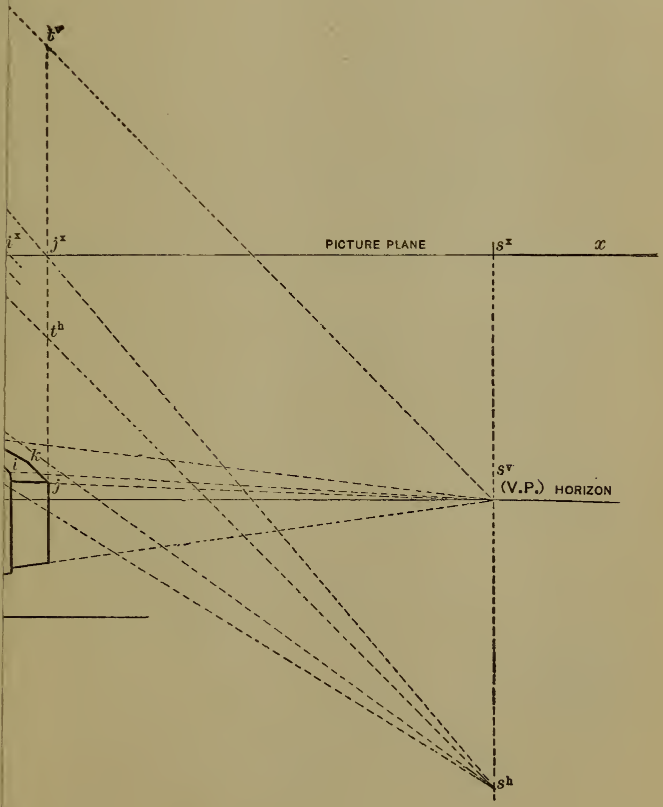


Fig. 3.

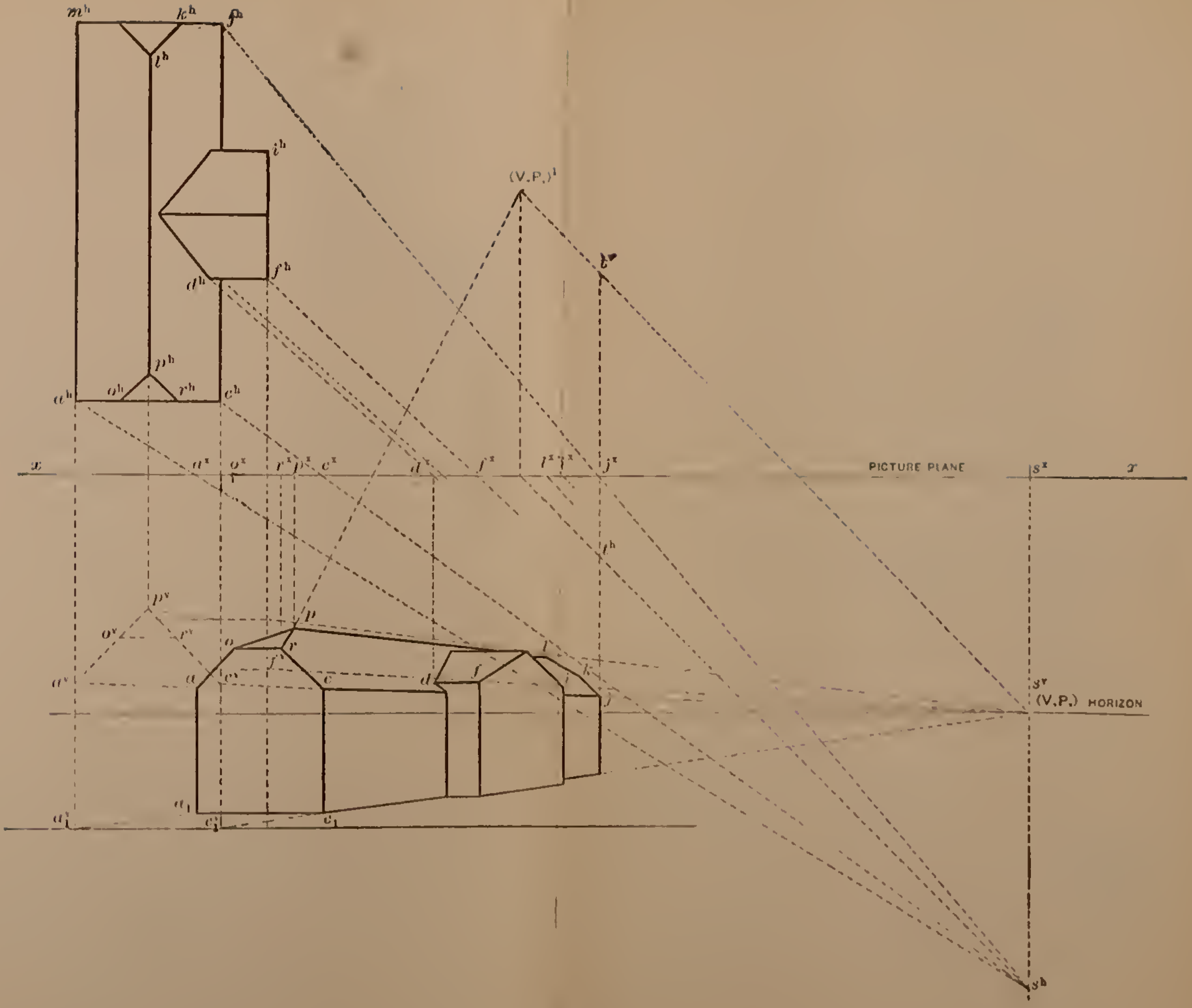

m^h
k^h
j^h
l^h
i^h
f^h
d^h
p^h
a^h
o^h
r^h
c^h
(V.P.)^1
l^v
x
a^x
o^x
r^x
p^x
c^x
d^x
f^x
l^x
i^x
j^x
PICTURE PLANE
s^x
x
l^h
p^v
o^v
r^v
a^v
p
o
r
a
f
c
d
f
l
k
j
s^v
(V.P.) HORIZON
a_1
c_1
c_1
s^h

Fig. 3.

Chapter III

ANGULAR PERSPECTIVE

VII. Figure 4 represents the angular perspective of a cube with a parallelopiped block resting against it. The object is placed entirely back of the picture plane and at any convenient distance from it, and according to the definition of angular perspective the vertical faces are placed at oblique angles with the picture plane as shown in the plan $a^hb^hc^he^h$. The vertical projection is shown by $a^vb^vc^ve^v$ and the end projection by $a^eb^ef^eg^e$.

The station S is selected at random so as to get as good a perspective as possible, its horizontal projection being denoted by s^h and its vertical by s^v on the horizon, which latter is the intersection of the picture plane with a horizontal plane through the station or point of sight. The vanishing points of the four systems of lines, 1st, $B\,C$, $G\,K$, etc.; 2d, $A\,B$, etc.; 3d, $E\,G$, etc.; 4th, $E\,F$, $I\,K$, etc.; are found by passing through the station lines parallel to these systems, as $s^hs^x{}_1$, parallel to b^hc^h and s^v (V. P.)^1 parallel to b^vc^v; $s^hs^x_2$ parallel to a^hb^h and s^v (V. P.)^2 parallel to a^vb^v; $s^hs^x{}_2$

parallel to e^hg^h and s^v (V. P.)^3 parallel to e^vg^v; $s^hs^x_2$ parallel to i^hk^h and s^vo^v parallel to i^vk^v,* respectively; and where they intersect the picture plane are the vanishing points of the respective systems, as (V. P.)^1; (V. P.)^2; (V. P.)^3, etc. The first two systems are horizontal lines and hence the vanishing points must be on the horizon. The last two systems are oblique lines, and the vanishing point must be above or below the horizon according as to whether the system of lines recede from the picture plane in ascending or descending. In this perspective the horizontal projections of the rays s^ha^h, etc., are drawn as in parallel perspective, intersecting the $X X$ axis or horizontal projection of the picture plane in a^x, b^x, e^x, etc. From these points perpendiculars to $X X$ are drawn and on these lines somewhere are the perspectives of the points A, B, E, etc. In order to locate these points we must first find the indefinite perspective of lines containing these points and the most convenient lines will be horizontal lines. Therefore pass a horizontal line through B, being one of the lines of the system $B C$, $G K$, etc., and hence coinciding with $B C$. The indefinite line $B C$ extends from the picture plane to infinity. Its intersection with P. P.

* If straight lines are parallel in space their corresponding projections are parallel.

being found by drawing a horizontal line through $b^h c^h$ until it intersects the $X\,X$ axis at n^x, then erect a perpendicular to $X\,X$, also draw a horizontal line through b^e, and where it intersects the perpendicular is the point where $B\,C$ intersects the picture plane denoted by n^v. This perpendicular $n^x n^v$ is called a measuring line because all lines in the face of the cube $B\,C$ intersect the P. P. in this line $n^x n^v$. Therefore the indefinite perspective of all horizontal lines lying in face $B\,C$ are drawn from points in $n^x n^v$ or $n^x n^v$ produced to (V. P.)^1, and the indefinite perspective of the line $B\,C$ is represented by n^v (V. P.)^1, and the definite perspective of the line $B\,C$ is found as in parallel perspective and shown by bc. The perspective of the bottom edge of the cube parallel to $B\,C$ is found exactly like $b\,c$. The line $A\,B$ being horizontal and of the second system we know that its vanishing point is (V. P.)^2, and the perspective of B is b, therefore the perspective of $A\,B$ must be on the line b (V. P.)^2, and the perspective of a is located as at c. The perspective of the cube is completed by the repetition of the methods described in this article or those under parallel perspective. The perspective of the parallelopiped is now to be described. By passing horizontal lines through E, F, and G, they intersect the picture plane in the points

r^v_1, r^v_{11}, r^v, respectively, and the line $r^h r^v_{11}$ is a measuring line for all heights in the plane $F\ G$. The indefinite perspective of these horizontal lines are r^v_{11} (V. P.)^2, r_1 (V. P.)^2, r (V. P.)^2, since they are of the system $A\ B$, etc. The perspectives e and f are found in the same manner as a and c. The perspective of $E\ G$, etc., is of the third system, and its vanishing point is (V. P.)^3. Its indefinite perspective is found by finding where $E\ G$ produced intersects the picture plane as $v\ i$ and connecting it with (V. P.)^3. On this indefinite perspective $v\ i$ (V. P.)^3 are the perspectives of E and G. E having already been found at e where the perpendicular from e^x intersects the indefinite perspective r^v_1 (V. P.)^2 which in turn lies on the indefinite perspective $v\ i$ (V. P.)^3. The perspective g of the point G is found from $g^h s^h$ and a perpendicular to $X\ X$ from g^x, and where it intersects the indefinite perspectives r^v (V. P.)^2 and $v\ i$ (V. P.)^3. The perspectives of the other lines of the third system are found in a similar manner, and the points e and f; i and k, etc., are connected and the perspective ei is completed. As a check the lines ef, ki, etc., of the fourth system should meet at (V. P.)^4. Another check is to find the perspectives of g, k, i, etc., as e and f were found, and see if the lines joining the proper points vanish to (V. P.)^3 or (V. P.)^4. By following the above method the entire perspective can be completed.

PROBLEMS IN ANGULAR PERSPECTIVE

VIII. 1. Find the perspective of Fig. 3 when one edge of the building is behind the picture plane, and find the vanishing point of at least one system of oblique lines.

2. Find the perspective of Fig. 3 when one edge is in the picture plane.

3. Find the perspective of Fig. 3 when the wing is removed and the end of the roof has no triangular sloping surface. One edge is behind the picture plane and the point of sight to the right of the object, find the vanishing point of at least one system of oblique lines.

4. Find the perspective of Fig. 3 when the wing is removed and the end of the roof has no triangular sloping surface. One edge behind the picture plane and the point of sight to the left of the object, and find the vanishing point of at least one system of oblique lines.

5. Find the perspective of Fig. 4 when the vertical edge through *B* is in the picture plane.

6. Find the perspective of Fig. 4 when block *EI* rests on edge *A B* and *E* is in the picture plane, and find the vanishing point of at least one system of oblique lines.

7. Find the perspective of Fig. 4 when block *EI* rests on edge *A B* and *E* is behind the picture plane.

8. Find the perspective of Fig. 4 when block *EI* rests on edge *C D* and vertical edge through *B* is behind the picture plane, and find the vanishing point of at least one system of oblique lines.

9. Find the perspective of Fig. 4 when block *EI* rests on edge *C D* and vertical edge through *B* is in the picture plane.

10. Find the perspective of Fig. 4 when block *EI* rests on edge *B C* and vertical edge through *B* is parallel to the picture plane. The whole object being behind the picture plane, find the vanishing point of at least one system of oblique lines.

11. Find the perspective of Fig. 4 when block *EI* rests on edge *B C* and its corner nearest to the picture plane in problem 10 is in the picture plane in this problem, and find the vanishing point of at least one system of oblique lines.

12. Find the perspective of Fig. 4 when the entire object is in front of the picture plane, and find the vanishing point of at least one system of oblique lines.

13. Find the perspective of a right parallelopiped when the object rests on its end one edge being in the picture plane.

14. Find the perspective of an oblique parallelo-

piped when the object rests on its end one edge being behind the picture plane, and find the vanishing point of at least one system of oblique lines.

15. Find the perspective of a right parallelopiped when the object rests on one of its faces and one edge is behind the picture plane and the end perpendicular to the base.

16. Find the perspective of a right octagonal prism when one edge is in the picture plane and one end is the base, and find the vanishing point of at least one system of oblique lines.

17. Find the perspective of a right octagonal prism when one corner is in the picture plane and one face is the base.

18. Find the perspective of a right octagonal prism when one edge is behind the picture plane and one edge is in the base, and find the vanishing point of at least one system of oblique lines.

19. Find the perspective of a right octagonal prism when one corner is behind the picture plane and one face is the base, and find the vanishing point of at least one system of oblique lines.

20. Find the perspective of a right octagonal prism when no faces are parallel to the picture plane and the octagon is the base, one of the edges being parallel to the picture plane, and find the vanishing point of at least one system of oblique lines.

21. Find the perspective of a parallelopiped block with a mortise in one face and one end of the solid is halved, one edge being parallel to the picture plane.

22. Find the perspective of a parallelopiped block when one end has a tenon and the other end square, one edge being parallel to the picture plane.

23. Find the perspective of a right square pyramid, one corner behind the picture plane. Find the vanishing point of at least one system of oblique lines.

24. Find the perspective of a right square pyramid when one corner is in the picture plane.

25. Find the perspective of an oblique square pyramid, and the vanishing point of one system of oblique lines.

26. Find the perspective of an oblique octagonal pyramid, and the vanishing point of at least one system of oblique lines.

27. Find the perspective of a tee made of two parallelopipeds of the same size and with square cross-sections, one edge in the picture plane.

28. Find the perspective of a tee made of two parallelopipeds of the same size and with square cross-sections, one edge being behind the picture plane.

29. Find the perspective of the frustum of an

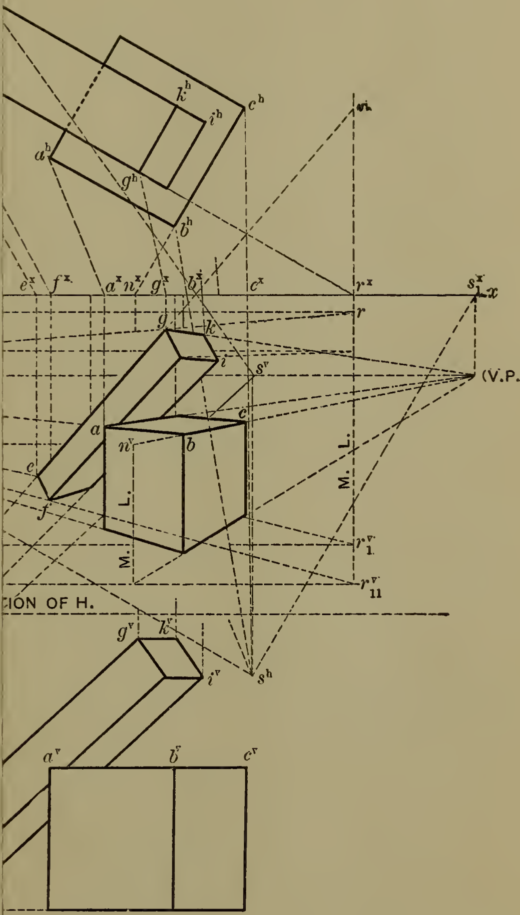
oblique square pyramid. Find the vanishing point of one system of oblique lines.

30. Find the perspective of a triangular prism and the vanishing point of one system of oblique lines.

Chapter IV

PERSPECTIVE OF CURVES

IX. The perspective of a curve is found by the methods of parallel or angular perspective. In general the former will be the simpler method as shown in Fig. 5. In this figure the curve chosen to find the perspective of is a circle whose horizontal projection is shown as $a^hb^he^hg^h$, etc., and vertical projection as a^ve^v. The picture plane, or $X X$ axis, horizon, station, and vanishing point are located as described in parallel perspective. The station having been determined we pass rays from it to a finite number of points on the curve, their horizontal projection being denoted by s^ha^h; s^hb^h; s^he^h; s^hf^h, etc., always being sure to draw the extreme rays $S D$ and $S I$ whose horizontal projections, s^hd^h and s^hi^h, are tangent to the curve. These projections of the rays cross the $X X$ axis in a^x, b^x, e^x, f^x, d^x, and i^x, respectively. Through the points A, B, E, etc., of the curve from which rays are drawn, pass horizontal lines perpendicular to the picture plane, their horizontal projections being shown by lines through a^h, b^h, e^h, etc.,



k^h
i^h
c^h
a^h
g^h
b^h
e^x
f^x
a^x
n^x
g^x
b^x
c^x
r^x
s_1^x
x
r
g
k
i
s^v
(V.P.)^1
c
a
n^v
b
e
f
L.
M.
M. L.
r_1^v
r_{11}^v
ION OF H.
g^v
k^v
i^v
s^h
a^v
b^v
c^v

G. 4.

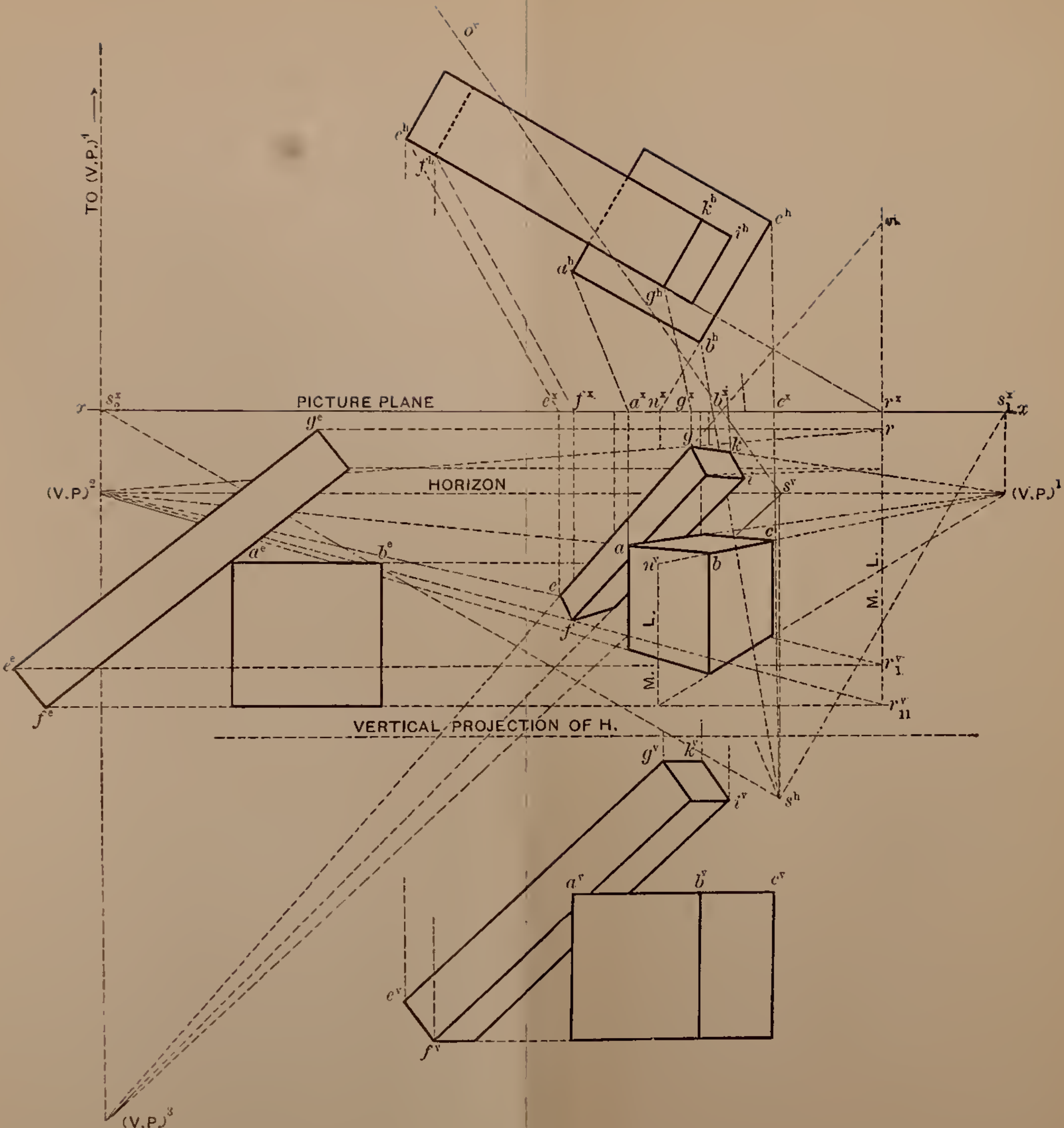

TO (V.P.)1 →
PICTURE PLANE
HORIZON
VERTICAL PROJECTION OF H.
(V.P.)2
(V.P.)1
(V.P.)3
M. L.
L.
M.
x
x
o
e
f
a
b
c
g
k
i
n
r
s

FIG. 4.

perpendicular to the $X X$ axis. Their front projections are shown in the points a^v, b^v, e^v, etc., respectively. The vanishing point of these horizontal lines is V. P., which coincides with s^v. The indefinite perspective of these horizontal lines is a^vV. P.; b^vV. P.; e^vV. P., etc., respectively. From the points a^x, b^x, e^x, etc., on the $X X$ axis draw lines perpendicular to the same, and where they intersect the indefinite perspectives a^v V. P.; b^v V. P.; e^vV. P., etc., respectively, in a, b, e, etc., is the perspective of these points of the curve. By this method as many points on the curve as are necessary can be obtained.

X. If we wish to find the perspective of a right cylinder as shown in Fig. 5, find the perspective of another curve whose horizontal projection coincides with the first curve and whose vertical projection is $a_1^v e_1^v$, in the same manner as in the first curve, as $a_1 b_1 e_1$, etc. In order to complete the perspective of the cylinder, vertical lines have to be drawn from the extreme points of the perspectives of the circles as ii_1, and dd_1. These points are the perspectives of the points I, D, I_1, D_1, and in horizontal projection are the points i^h, d^h, where the rays are tangent to the curve. By this method the perspective of any curved object can be obtained.

PROBLEMS IN PERSPECTIVE OF CURVES

XI. 1. Find the perspective of a right circular cone.

2. Find the perspective of a right frustum of a circular cone.

3. Find the perspective of an oblique cylinder with an elliptical base.

4. Find the perspective of an oblique frustum of a cone whose right section is an ellipse.

5. Find the perspective of an oblique cone whose right section is a circle.

6. Find the perspective of the intersection of a right circular cylinder with an oblique cone having its right section a circle.

7. Find the perspective of a sphere.

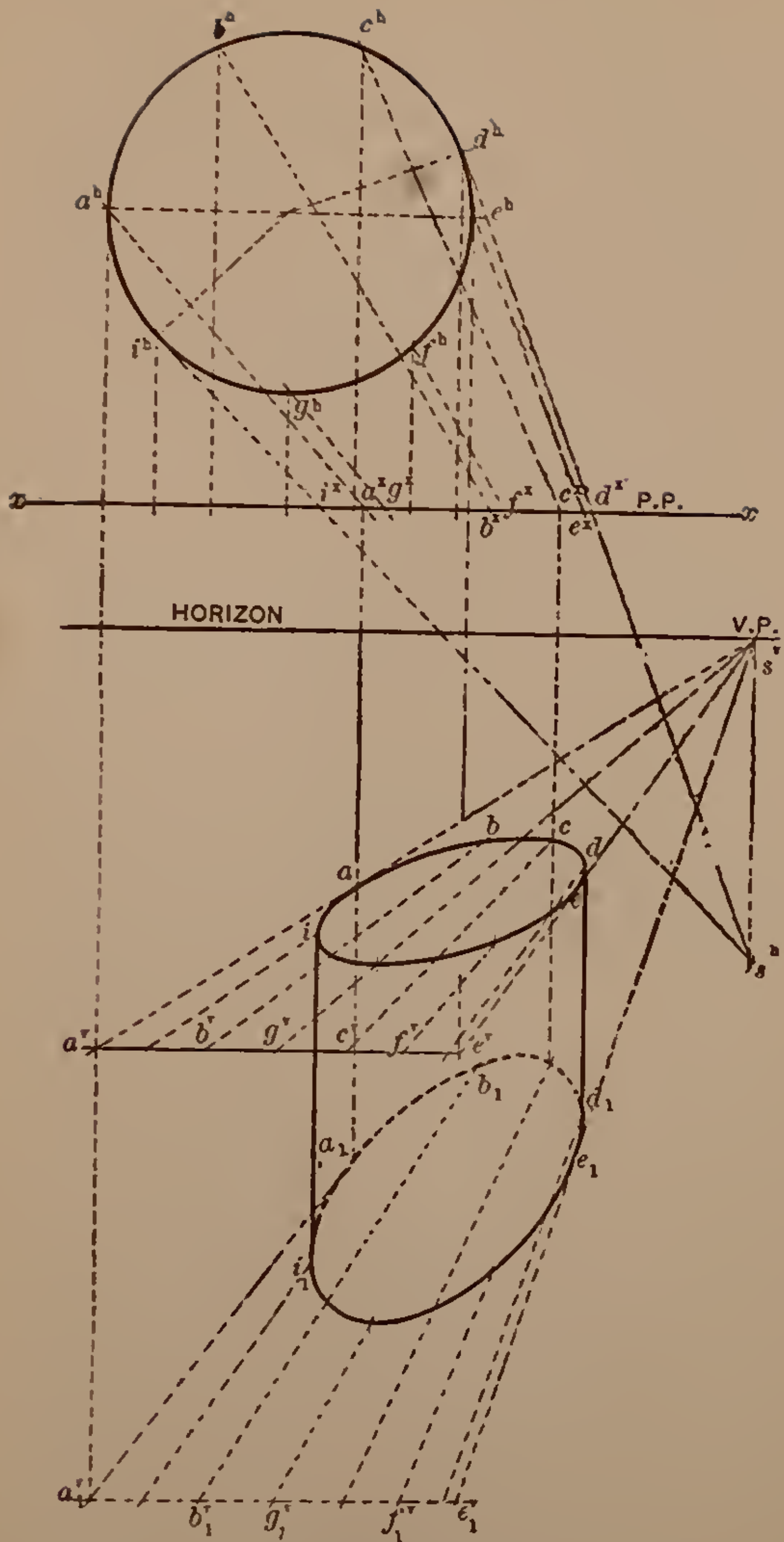

b^h
c^h
d^h
a^h
e^h
i^h
f^h
g^h
x
i^x
a^x
g^x
f^x
c^x
d^x
P.P.
x
b^x
e^x
HORIZON
V.P.
s^v
b
c
a
d
i
e
s^h
a^v
b^v
g^v
c^v
f^v
e^v
b_1
d_1
a_1
e_1
i_1
a_1^v
b_1^v
g_1^v
f_1^v
e_1^v

FIG. 5.

Chapter V

PERSPECTIVE OF SHADOWS

XII. Fig. 6 represents the perspective $ga_sb_sc_se$ of the shadow of the frustum of a square pyramid. The perspective $abcg$ of the object is first drawn by the methods already outlined in this part. The shaded portion $g^ha_s^hb_s^hc_s^h,e^h$ is the horizontal projection of the shadow of the frustum on the horizontal plane of its base, and is drawn by the use of the horizontal and vertical projections of the rays of light aided by the plan $a^hb^hc^hg^h$ and the elevation $a^vb^vc^vg^v$.

The perspective $ga_sb_sc_se$ of this shadow $g^ha_s^hb_s^hc_s^he^h$ and the method of obtaining it will now be explained. Since the orthographic projection of this shadow is in the horizontal plane all of its bounding lines, $g^h\ a_s^h$; $a_s^hb_s^h$, etc., and the horizontal projections of the rays $a_1{}^ha_s^h$; $b_1{}^hb_s^h$, etc., are all horizontal lines, and therefore have their vanishing points in the horizon.

Since the horizontal projections of all the rays $a_1{}^h\ a_s^h$; $b_1{}^hb_s^h$, etc., are parallel they will have the same vanishing point. This is obtained by passing through the station a line parallel to the system of lines $a_1^h a_s^h$; $b_1^h b_s^h$, etc., and where it intersects the picture plane is the vanishing point of that system.

We therefore pass through the horizontal and vertical projections of the station s^h and s^v, respectively, a line parallel to the respective projection of the system of lines $a_1^h a_s^h$; $b_1^h b_s^h$, and obtain by descriptive geometry the vanishing point (V. P.)^4 of this system. The indefinite perspective of the lines $a_1^h a_s^h$; $b_1^h b_s^h$, etc., is a_1 (V. P.)^4; b_1(V.P.)^4, etc., respectively. The definite perspectives of these lines respectively are $a_1 a_s$; $b_1 b_s$, etc., found by drawing horizontal projections of rays $a_s^h s^h$; $b_s^h s^h$, etc., and where they intersect the $X X$ axis in a_x^h; b_x^h, etc., drop perpendiculars, and where they intersect the indefinite perspective of the corresponding lines we get respectively a_s, b_s, etc. To complete the perspective join g, a_s; a_s, b_s; b_s, c_s and c_s, e, and we have the shaded portion as the visible portion of the perspective of the shadow of the frustum of the pyramid.

As a check to this perspective, find the vanishing points (V. P.)^2 for line $a_s^h b_s^h$, which is parallel to line $a^h b^h$, etc., and hence has the same vanishing point; (V. P.)^5 for line $g^h a_s^h$; (V. P.)^1 for line $c_s^h b_s^h$, which is parallel to line $c^h b^h$, etc., and hence has the same vanishing point; (V. P.)^3 for line

$e^h c^h_s$. With these vanishing points found draw the indefinite perspective g (V. P.)^5, and as a check it should pass through a_s; next through a_s draw the indefinite perspective a_s (V. P.)^2, and as a check it should pass through b_s, again the indefinite perspective b_s (V. P.)^1 should pass through c_s and the indefinite perspective e (V. P.)^3 should also pass through c_s.

The perspective could have been drawn without the use of the lines $s^h c^h_s$, $s^h b^h_s$, etc., by using only the five vanishing points. The intersection, of the indefinite perspectives a_1 (V. P.)^4, g (V. P.)^5, in a_s gives one point in the perspective, a_s (V. P.)^2, b_1 (V. P.)^5, in b_s gives a second, b_s (V. P.)^1, c_1 (V. P.)^5 in c_s gives a third point, and c_1 (V. P.)^5, e (V. P.)^3 should also give c_s.

XIII. The best method and the one that takes the least number of lines is the method by the vanishing point of rays, and will now be described. It consists of finding the vanishing point of the horizontal projections of the rays of light and the vanishing point of the light rays themselves. The orthographic projection of the shadow in the horizontal plane is entirely ignored and has no use in this method. The horizontal projection of the rays of light make 45° with the $X X$ axis, and the vanishing point of these is found in (V. P.)^4 as already described. The vanishing point of the rays themselves is found by passing through s^h and s^v lines parallel to the horizontal and vertical projections of the rays, which according to conventionality are 45° and 315° respectively, with the $X\,X$ axis, and where it intersects the vertical or picture plane is the vanishing point of the rays themselves. This vanishing point is found at $(V.\ P.)^6$. The perspective will now be drawn. The indefinite perspective of the horizontal projection of the ray through a is a_1 $(V.\ P.)^4$, and the indefinite perspective of the ray itself is a $(V.\ P.)^6$. These two lines meet in a_s, and this is where the ray itself intersects the horizontal plane, and is therefore the perspective of the shadow of a in the horizontal plane. The indefinite perspective of the horizontal projection of the ray through b is b_1 $(V.\ P.)^4$, and the indefinite perspective of the ray itself is b $(V.\ P.)^6$. These two lines intersect in b_s, and this is where the ray itself intersects the horizontal plane and is therefore the perspective of the shadow of b in the horizontal plane. In the same manner the shadow of all points of the object is found. To complete the shadow join g, a_s; a_s, b_s; etc., for all points of the shadow found. Thus we get as our complete shadow $ga_sb_sc_se$ of the frustum and the shaded portion is the visible part of the perspective of the shadow.

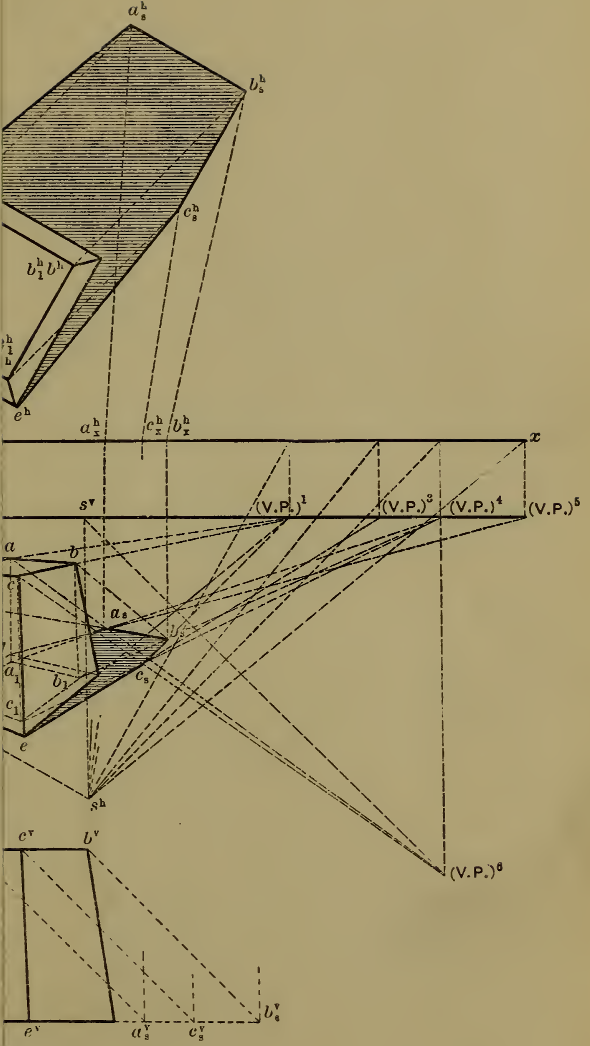

a_s^h
b_s^h
c_s^h
$b_1^h b^h$
b_1^h
e^h
a_x^h
c_x^h
b_x^h
x
s^v
$(V.P.)^1$
$(V.P.)^3$
$(V.P.)^4$
$(V.P.)^5$
a
b
c
a_s
b_s
a_1
b_1
c_s
c_1
e
s^h
c^v
b^v
$(V.P.)^6$
e^v
a_s^v
c_s^v
b_s^v

FIG. 6.

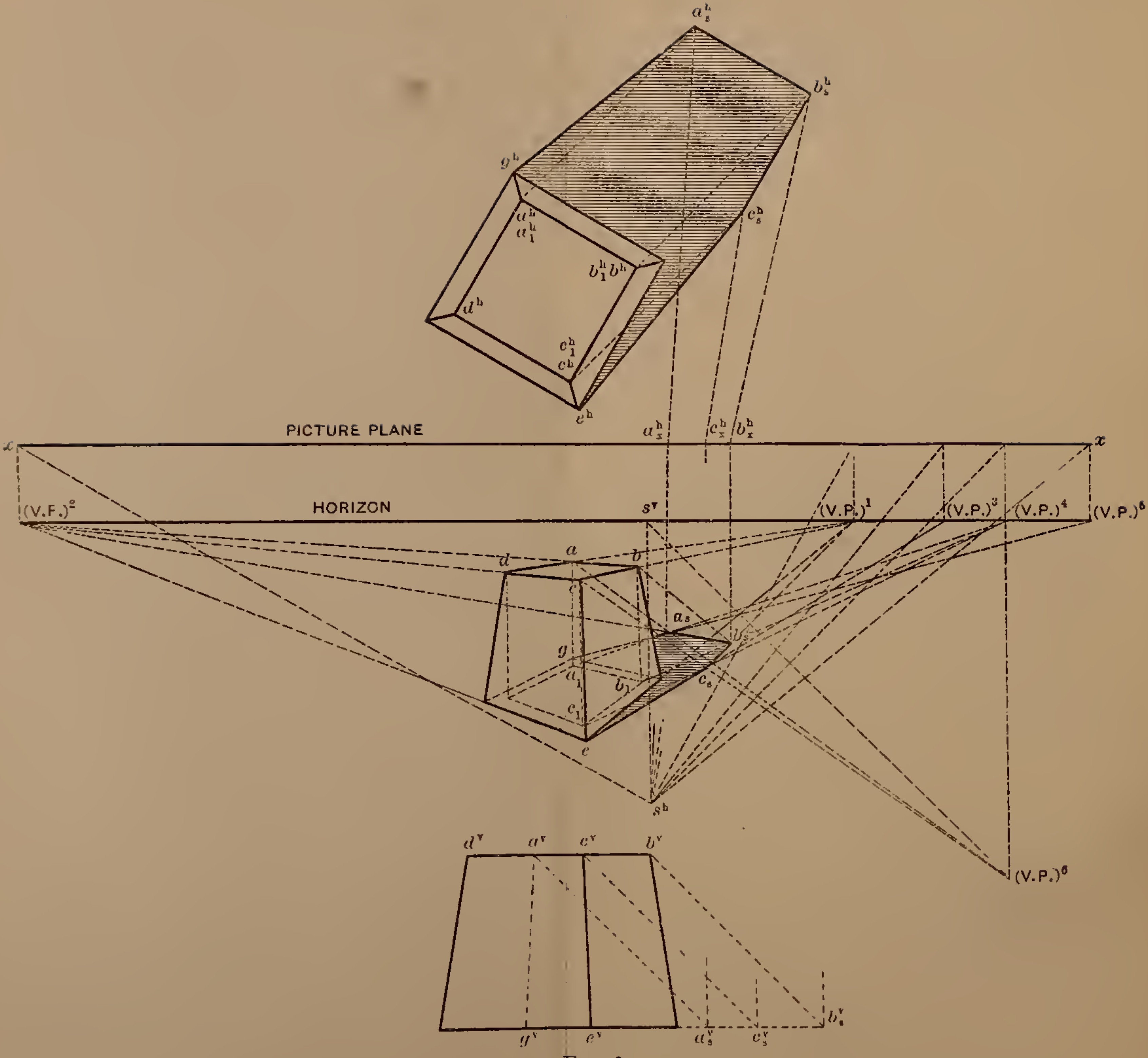

a_s^h
b_s^h
g^h
a^h
a_1^h
c_s^h
b_1^h b^h
d^h
c_1^h
c^h
e^h
PICTURE PLANE
a_s^h
c_s^h
b_s^h
x
x
HORIZON
(V.F.)^2
s^v
(V.P.)^1
(V.P.)^3
(V.P.)^4
(V.P.)^5
d
a
b
c
a_s
b_s
g
a_1
b_1
c_s
c_1
e
s^h
(V.P.)^6
d^v
a^v
c^v
b^v
g^v
e^v
a_s^v
c_s^v
b_s^v

Fig. 6.

XIV. Fig. 7 shows the reproduction of the perspective of a cube and a parallelopiped block, resting on one edge of the cube, carefully described Fig. 4. In addition to this is shown the perspective of the shadow of the object by the method of the vanishing point of rays, and the description will be confined to the details of this method. $(V. P.)^3$ is the vanishing point of the horizontal projection of the rays and $(V. P.)^4$ is the vanishing point of the rays themselves. The perspective of the horizontal projection of the ray through b is b_1 $(V. P.)^3$, and the perspective of the ray itself is b $(V. P.)^4$, these rays meet in b_s and is the perspective of the shadow of b, and b_1b_s is the perspective of the shadow of bb_1.

The perspective of the shadow of c is found at c_s by the same method that b_s was found, and b_s c_s is the perspective of the shadow of bc. The line cd is parallel to the horizontal plane, and hence the vanishing point of its shadow must be the same as the vanishing point of the line. Therefore c_s $(V. P.)^2$ is the indefinite perspective of the shadow of dc. The perspective of the shadow of dd_1 is found exactly as bb_1, and its indefinite perspective is d_1 $(V. P.)^3$. This gives the complete shadow of the cube, but on account of the parallelopiped resting on the cube there is a certain portion of the cube that does not cast a shadow, and that portion of the cube that does not cast a shadow, the parallelopiped will cast a shadow on it.

The horizontal projection of the point n is n_1, found by passing a vertical plane through fn and (V. P.)^2, intersecting the horizontal plane in the line f (V. P.)^2. This same vertical plane intersects the face db of the cube in the line l (V. P.)^2, l being the point where the edges fn and ad intersect. From n draw a line perpendicular to $X X$ axis and intersecting line f (V. P.)^2 and the horizontal plane in n_1. This same perpendicular intersects the line l (V. P.)^2 and the horizontal plane db of the cube in the point n_1^1. The horizontal projection of the ray through n is n_1 (V. P.)^3, and the same projection of the ray on the plane db of the cube is n_1^1 (V. P.)^3. The perspective of the ray itself is n (V. P.)^4, and intersects n_1 (V. P.)^3 and n_1^1 (V. P.)^3 in n_s and n_s^1, respectively, which are the perspectives of the shadow of the point n on the horizontal plane of the base of the cube and the plane db of the face of the cube respectively. The perspectives of the shadow of f and l are f and l themselves, since they are in planes that the perspective is in. The perspective of the shadow of the line fn would be fn_s if there were no cube to cast a shadow on, but since that is the case, the shadow falls on the horizontal plane in two different parts and on two faces of the cube, namely, ad_1 and db. The first part of the line fn considered is fl and the perspective of its shadow is that portion of fn_s between f and the point r_s where it intersects the perspective d_1 (V. P.)^3 of the shadow of the edge dd_1, and that part lying in the face ad_1 of the cube. This shadow lying on the cube is found by extending d_1 (V. P.)^1 to fr_s, and the former intersects the latter in h_s. Then h_s lies on the face ad_1 produced and h_s is a shadow of some point in fn. Therefore the portion of the shadow of fn on the indefinite plane ad_1 is lh_s and it intersects dd_1 in r_s^1. The shadow lying on the cube is hence lr_s^1. The point casting the shadow r_s^1 also casts the shadow r_s, for the perspective of the ray through r_s^1 is r_s^1, (V. P.)^4 and it passes through r_s. Hence the perspective of the shadow of fl is fr_s and $r_s^1\, l$. The second portion of the line fn is ln, and the perspective of the shadow lies on the face db of the cube and that portion of the line fn_s between n_s the perspective of the shadow of n and the intersection o_s of the perspective fn_s and c_s (V. P.)^2. The part on the face db is found by joining ln_s^1 and that portion of the line lo_s^1 included between the sides ad and dc in the perspective of the shadow of the line fn on the face db. The remainder of the line

$o_s^1\ n_s^1$ has the perspective of its shadow in the line $o_s n_s$ on the horizontal plane. The perspective of the shadow of the line fn is now completed, and is represented by fr_s, $r_s^1 l$; $l\ o_s^1$; and $o_s n_s$. The line shown joining r_s and r_s^1, also o_s^1 and o_s are in the perspective of the same ray of light, it being the point where the perspective is just leaving one surface and entering upon another.

The next point is to determine what lines in the end $g\,n\,i\,k$ cast shadows on the horizontal plane of the base of the cube. This is determined by finding which of the four lines have their shadow the farthest to the right. There can be only two of them that cast a shadow and these are found to be ni and ik. The perpendiculars from i and k are found the same as for n, and in the case of k is shown as k_1. The perspectives of the shadows of i and k are found exactly as for n, and are shown at i_s and k_s. The perspectives of the shadows of ni and ik are hence $n_s\,i_s$ and $i_s\,k_s$. The next element of the parallelopiped that casts a shadow is mk. The shadow of k has been found and is k_s, while that of m is m_s, and found in the same way as k_s. The shadow of mk is hence $m_s\,k_s$. This shadow falls outside of the shadow of d, hence this line $m_s\,k_s$ is the limit of the shadow in this direction. The shadow of the end $f\,e\,m$ is next found, and since this plane is parallel to the plane *g k i n*, the lines casting the shadow will be the reverse of those casting the shadow in plane *g k i n*. In *g k i n*, *ni* and *ik* cast shadows while in plane *f e m*, *fe* and *em* must be the lines casting shadows. The shadow of these latter two lines are found in exactly the same manner as the former two lines. The entire perspective of the shadow and of the cube and parallelopiped have been found and the visible portions of this perspective have been cross-hatched. The outline of the shadow has been lettered at almost every angle, and those points that have been lettered will be mentioned to show clearly the outline of the perspective. They are as follows: — b_1, b_s, c_s, o_s, n_s, i_s, k_s, m_s, e_s (not shown), f, r_s, d_1, c_1 (not shown), and b_1 in the horizontal plane, while on the cube they are r_s^1, l, o_s^1, d, r_s^1.

XV. Fig. 8 contains a given oblique plane *W* whose traces are H. W. and V. W., *i.e.*, horizontal and vertical traces * of the plane *W* respectively, and the perspective of the frustum of the pyramid the same as shown in Fig. 6 being an object in angular perspective.

The horizontal and vertical projections of the frustum are shown as $a^h b^h c^h d^h e^h g^h$ etc., and $a^v b^v c^v d^v e^v g^v$ etc., respectively, and its perspective by $a b c d e g$, etc. The method of drawing this perspective has been described in previous articles under angular perspective, and the vanishing points of the horizontal lines are shown as $(V. P.)^1$ and $(V. P.)^2$. Under previous articles on the perspective of shadows it was shown how to find the vanishing point of the horizontal projection of rays and also the vanishing point of the rays themselves, and their vanishing points in Fig. 8 are $(V. P.)^3$ and $(V. P.)^4$ respectively.

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* By traces of a plane are meant the lines of intersection of the plane with the planes of projection, *i.e.*, in perspective horizontal and picture plane.

The problem in Fig. 8 is to find the perspective of the shadow of this frustum $A B C D E G$, etc., on the given oblique plane W. It can be found in two ways, 1st, by finding the orthographic projections of the shadow on the plane W and then find the perspective of the shadow from these projections. 2d, by finding the perspective of the shadow on the oblique plane W direct without the use of the horizontal and vertical projections of the shadow, which will be the method now described and illustrated in this figure.

Since the conventional direction of rays is such that their horizontal and vertical projections make 45° and 315° with the $X X$ axis respectively, we have projections of these lines as c^h

(h. i.)^1, $c^v w^v$; e^h (h. i.)^2, e^v w^v_1, etc., respectively. Imagine vertical projecting planes *U*, *Y*, *Z*, etc., passed through these lines, and their horizontal and vertical traces shown by H. U., V. U.; H. Y., V. Y.; H. Z., V. Z., etc. Since the rays of light are parallel, these planes *U*, *Y*, *Z*, etc., are parallel. These horizontal traces H. U., H. Y., H. Z., etc., intersect the horizontal trace H. W. of the plane *W* in (h. i.)^1; (h. i.)^2; (h. i.)^3, etc., respectively, and the vertical traces V. U., V. Y., V. Z., etc., of these same planes intersect that of *W* in (v. i.)^1; (v. i.)^2; (v. i.)^3, etc., respectively. These points hence lie in the horizontal and vertical planes of projection respectively. These planes *U*, *Y*, *Z*, etc., if produced far enough will intersect the given oblique plane *W* in straight lines which in space pass between the points (h. i.)^1, (v. i.)^1; (h. i.)^2, (v. i.)^2; (h. i.)^3, (v. i.)^3, etc., and whose projections are c^h (h. i.)^1, (v. i.)^1 k^v_x; e^h (h. i.)^2, (v. i.)^2 m^v_x; g^h (h. i.)^3, (v. i.)^3 o^v_x etc. These lines of intersection with plane *W* are all parallel and hence have the same vanishing point. To find the vanishing point of the intersecting lines between the vertical projecting planes and the plane *W* pass a line through the station parallel to these lines, and where it intersects the picture plane will be the vanishing point of this system of parallel lines. Therefore through the horizontal projection of the station s^h draw a line parallel to c^h (h. i.) ^{1}; e^h (h. i.) ^{2}; g^h (h. i.) ^{3}, etc., intersecting the XX axis in n^h_x. Now through the vertical projection of the station s^v draw a line parallel to (v. i.) ^{1} k^v_x; (v. i.) ^{2} m^v_x; (v. i.) ^{3} o^v_x, etc., and where it intersects a perpendicular to the XX axis through n^h_x is where this line through the station intersects the picture plane, as (V. P.) ^{5} is the vanishing point of the above system of lines. The lines of intersection between the planes U, Y, Z, etc., and the given oblique plane W intersect the picture plane in points (v. i.) ^{1}; (v. i.) ^{2}; (v. i.) ^{3}, etc., and the perspective of these points are the points themselves. The indefinite perspectives of these intersecting lines are therefore (v. i.) ^{1} (V. P.) ^{5}; (v. i.) ^{2} (V. P.) ^{5}; (v. i.) ^{3} (V. P.) ^{5}, etc., and of the rays themselves are c (V. P.) ^{4}; e (V. P.)^4; g (V. P.) ^{4}, etc. Since the rays of light themselves and these intersecting lines lie in the same planes they will intersect and their points of intersection will be in the plane W and hence the shadows of some points of the object. Therefore, the intersections of these two systems of indefinite perspectives will be the perspectives of the shadows of the corners of the frustum of the pyramid on the given oblique plane W.

We will now find the perspective of the shadow of the point C on the given oblique plane W. The horizontal projection of the ray of light through C is c^h (v. i.)^1, and the traces of the vertical projecting plane U containing this ray of light are H. U. and V. U. respectively. The horizontal and vertical projections of the line of intersection between planes U. and W are c^h (h. i.)^1, (v. i.)^1 k^v_x and the indefinite perspective of this line of intersection is (v. i.)^1 (V. P.)^5. On this indefinite perspective is the perspective of the shadow of the point C on the plane W. Also the shadow of the point C is on the ray through C and is where the ray intersects or pierces the plane W. The perspective of C is c and the indefinite perspective of the ray through C is c (V. P.)^4 which contains the perspective of the shadow of C on the plane W. Since the perspective of the shadow of C on the plane W is on both systems of indefinite perspectives it must be at the intersection of these two systems. Therefore the intersection c^w_s of the two indefinite perspectives (v. i.)^1 (V. P.)^5 and c (V. P.)^4 will locate the perspective of the shadow of C on the given plane W. By the same method we can get the perspective of all the points of the frustum of the pyramid that cast a shadow on the plane W.

In order to determine what points cast the shadow on the oblique plane *W*, we can find the perspective of the shadow of all corners of the frustum, and those that fall within the extreme or limiting points of the shadow will be obliterated by these extreme points. By a careful observation one is able to discover which points fall within the shadow boundary without the labor of finding the exact perspective of the shadow of each point on the plane *W*.

As a check on the perspective of the shadow of the frustum on the plane *W*, we can find the vertical projections of the points where the rays of light intersect the plane *W*. By finding the distance the points are below the horizontal plane of projection we can find the indefinite perspectives of horizontal lines containing these intersections and lying in vertical projecting planes containing the rays of light. These rays themselves and these horizontal lines intersect the plane *W* in the same point and therefore intersect each other at that point. The perspectives of these points of intersection of the rays and the horizontal lines give also the perspectives of the shadows of the points of the frustum on the oblique plane *W*.

Two of the checks have been shown in Fig. 8, but here will be described only one of them as follows: Draw the vertical projection of the horizontal plane showing how far this plane is above the frustum of the pyramid. This line corresponds to the $X X$ axis shown above and is lettered $X_1 X_1$. Produce k^v_x (h. i.)^1 until it intersects the axis $X_1 X_1$ in the point k^v_{x1}, and through this point draw $k^v_{x1} w^v$ parallel to k^v_x (v. i.)^1. This line is then the vertical projection of the line of intersection of the planes U and W. Now through the vertical projection c^v of the corner C of the frustum, draw the vertical projection $c^v w^v$ of the ray, and where it intersects the line $k^v_{x1} w^v$ in the point w^v is the vertical projection of the intersection of the ray through C with the plane W. This projection shows the true distance that the point W is below the horizontal plane, and a horizontal line passed through this point W intersects the picture plane at the same distance $w^v l^v_{x1}$ below the horizontal plane. We next lay off this distance $w^v l^v_{x1}$ below the $X X$ axis and on the vertical trace V. U. of the plane U, since this horizontal line through the point W lies in the plane U, and the point c^v_1 is located. Since this horizontal line lies in the plane U, its vanishing point will be (V. P.)^3 and the indefinite perspective of this line is c^v_1 (V. P.)^3 and should intersect the perspective of the ray c (V. P.)^4 in the same point c^w_s that the perspective of the line of intersection (v. i.)^1 (V. P.)^5 does the perspective of this ray. It is seen from Fig. 8

that this point c_s^w does coincide with the first position of c_s^w and that the check proves that there is no error in the first method of obtaining the perspective c_s^w of the shadow of the point C. In like manner the drawing shows the check on the perspective e_s^w of the shadow of the point E and also demonstrates that there is no error in locating this point. All the points of the perspective of the shadow could be checked by following the same method as just described, but what has been said will be sufficient to understand the principles underlying this check.

The lines casting the shadow will now be given in the order of sequence of connecting points as EC, CB, BA, AG, GF, FE, and the perspectives of the shadows of the ends of these lines, in the same order of sequence on the oblique plane W, are e_s^w, c_s^w, b_s^w, a_s^w, g_s^w, f_s^w. After these points are connected in the proper order it is seen that part of the shadow is out of sight, being behind the perspective of the object, and that the outline of this part is in dotted lines. The portion of the shadow that is visible is cross-hatched and its corners are f_s^w, e_s^w, c_s^w, b_s^w, a_s^w, while the invisible corner is g_s^w. Thus the complete perspective of the shadow of the frustum of a pyramid on an oblique plane is shown.

PROBLEMS IN PERSPECTIVE OF SHADOWS

XVI. 1. Find the perspective of the shadow cast by the object as shown in Fig. 2.

2. Find the perspective of the shadow cast by the object as shown in Fig. 3.

3. Find the perspective of the shadow of the object as shown in Fig. 5.

4. Find the perspective of the shadow of the object as given in question 18, page 14.

5. Find the perspective of the shadow cast by the object as given in question 25, page 15.

6. Find the perspective of the shadow cast by the object in question 26, page 15.

7. Find the perspective of the shadow cast by the object in question 27, page 15.

8. Find the perspective of the shadow of the object as given in question 29, page 16.

9. Find the perspective of the shadow of the object as given in question 2, page 28.

10. Find the perspective of the shadow of the object as given in question 5, page 28.

11. Find the perspective of the shadow of the object as given in question 6, page 28.

12. Find the perspective of the shadow of the object as given in question 7, page 28.

13. Find the perspective of the shadow of the object as given in question 1, page 21.

14. Find the perspective of the shadow of the object as given in question 4, page 21.

15. Find the perspective of the shadow of the object as given in question 6, page 21.

16. Find the perspective of the shadow of the object as given in question 16, page 23.

17. Find the perspective of the shadow of the object as given in question 22, page 24.

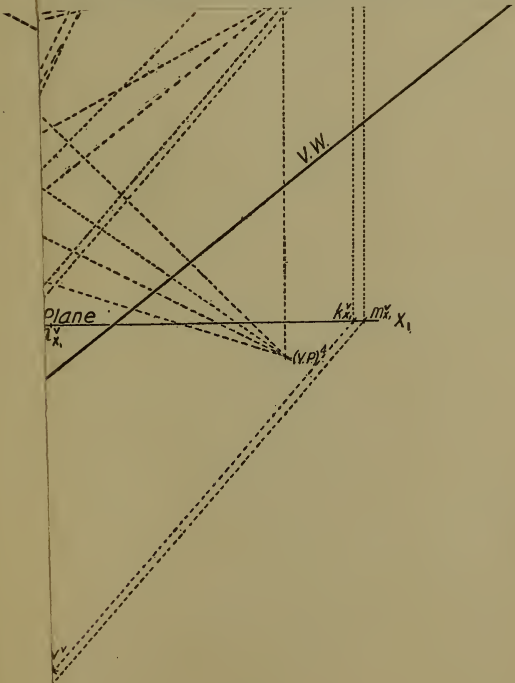
18. Find the perspective of the shadow of the object as given in question 25, page 24.

19. Find the perspective of the shadow of the object as given in question 27, page 24.

20. Find the perspective of the shadow of the object as given in question 29, page 24.

21. Find the perspective of the shadow on an oblique plane of the frustum of a right circular cone when its base is parallel to the horizontal plane.

Fig. 7.



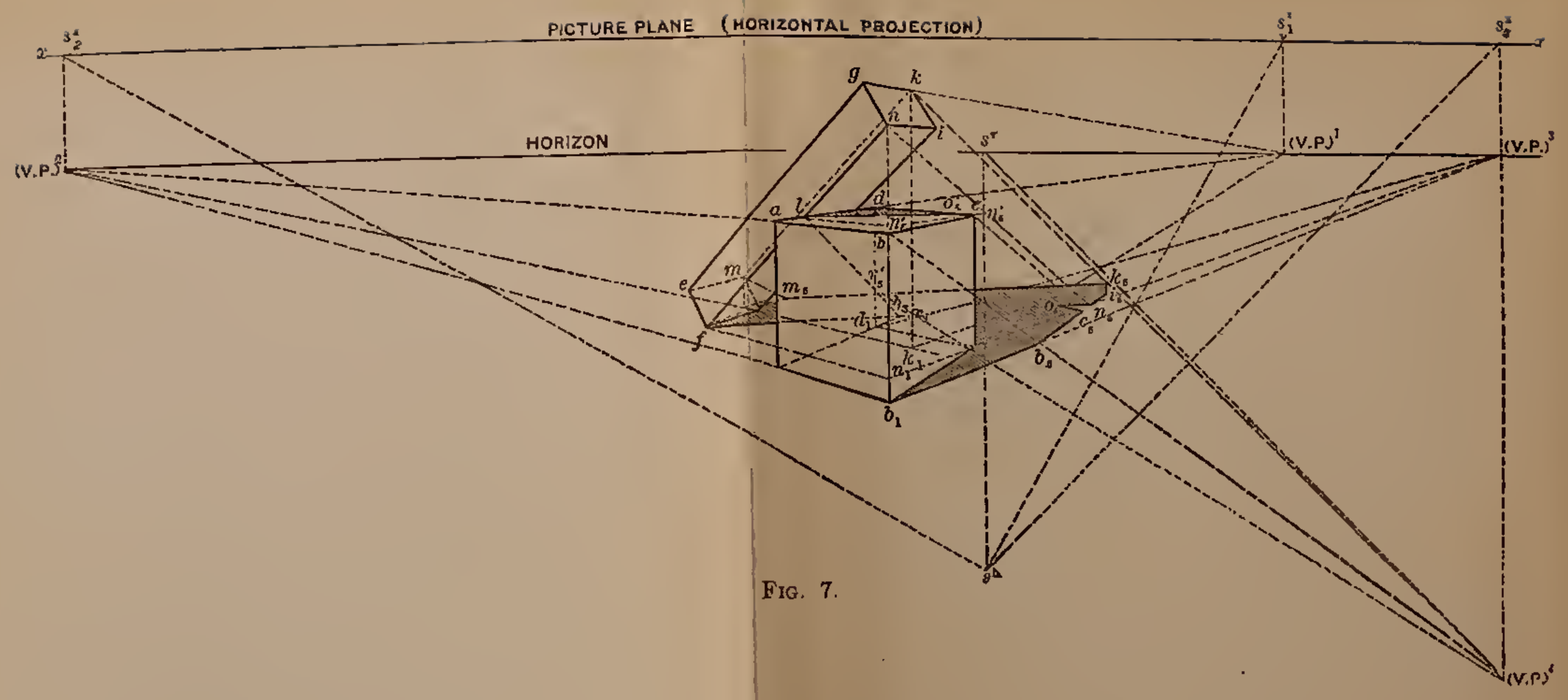

PICTURE PLANE (HORIZONTAL PROJECTION)
HORIZON
(V.P.)²
(V.P.)¹
(V.P.)³
(V.P.)⁴
g
k
h
l
a
d
b
e
f
m
b₁
b₂
s⁴

Fig. 7.

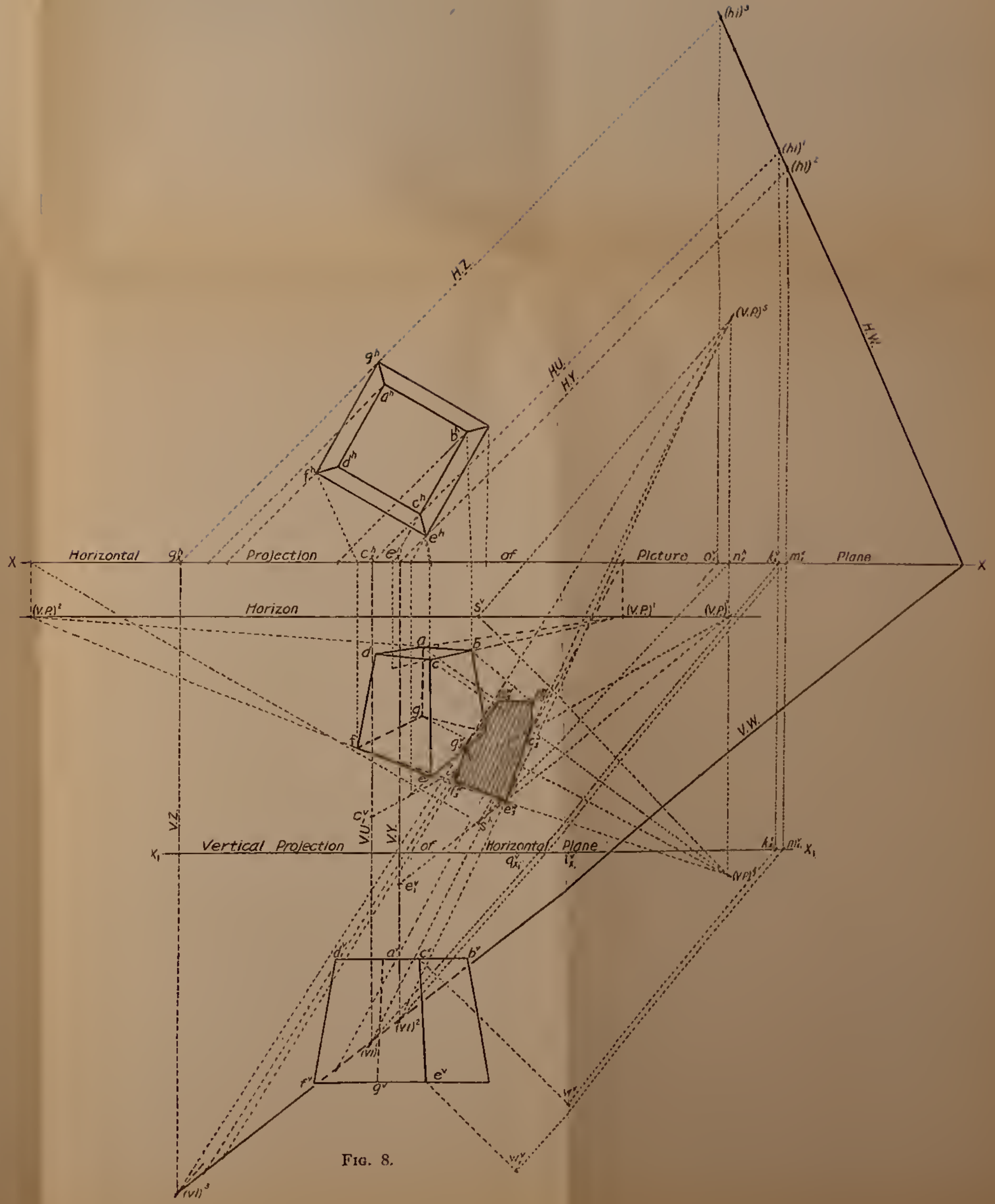

(hl)³
(hl)¹
(hl)²
H.Z.
H.U.
H.Y.
H.W.
(V.P.)³
Horizontal Projection of Picture Plane
X
Horizon
(V.P.)²
(V.P.)¹
(V.P.)
Vertical Projection of Horizontal Plane
X₁
V.Z.
V.U.
V.Y.
V.W.
(V.P.)⁴
(vl)²
(vl)
(vl)³
a
b
c
d
e
f
g

Fig. 8.

PART II

PERSPECTIVE BY THE METHOD OF SCALE

Chapter VI

PERSPECTIVE BY THE METHOD OF SCALE

XVII. The first article on perspective was written by using the orthographic projections of the object and the rays to obtain the perspective of the same. A pictorial illustration of this method is shown in Fig. 1, and the actual drawing of this method for that particular object is shown in Fig. 2.

The second article treats of the method by scale, and Fig. 10 shows this method for the same object as well as for the same position as was used in Fig. 2.

If the first method is used you have to place the plan, not necessarily draw it, upon the drawing-board. This as a rule requires a wide drawing-board and a long straight-edge to draw the horizontal projections of the rays. In this method you get both the perspective of points and lines of the object. This method usually requires fewer lines and less time to draw the perspective, after the plan and elevations are correctly placed, than the second method does.

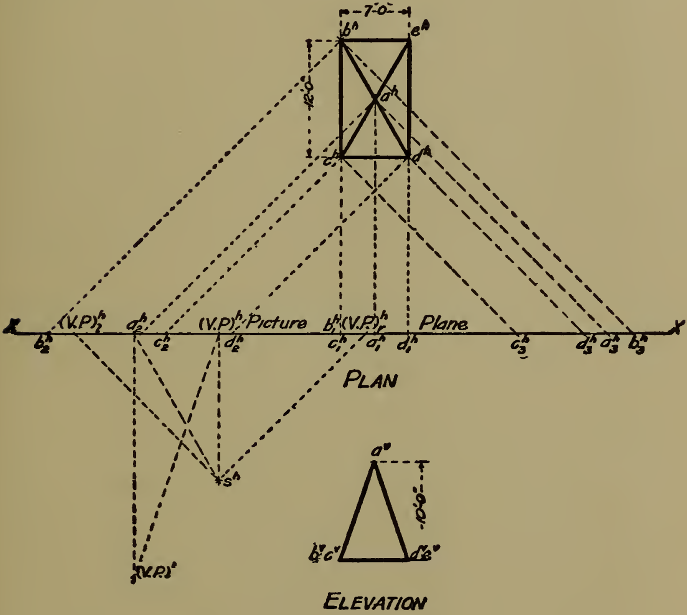
In the second method, as well as in the first method, the projections of the object are on different pieces of paper from those upon which the perspective is drawn. In the first method they are usually tacked to the drawing-board in their proper position for drawing the perspective. In the second method they simply lie loosely upon the drawing-board to be used only for scaling or computing distances. In this second method, the method by scale, the drawing-board does not have to be as large, and hence does not take so much room to draw it. In this method we very seldom draw the perspective of a line of the object, unless we do it as a check, it always being to find the perspective of a point. The perspective of a point being found by the intersection of the perspective of two horizontal lines, lying in the same plane as the given point, and making 45° with the picture plane. This shows that for the perspective of each point you must have the perspective of at least two lines, and the point where each one of these lines intersects the picture plane must be scaled in order to get their perspective. If you wish a check on this work you must have a third horizontal line in the same plane as the other two lines, and the point where this intersects the picture plane must also be located by scaling before its perspective can be drawn. This third or check line is usually perpendicular to the picture plane, and hence its vanishing point is in the center of picture. You can now see that the advantage of the second method is that it takes a smaller board than the first method, especially in width, while the length in each method is about the same. The disadvantage is that there is a great deal of scaling and the perspective of a great many construction lines, most of them being horizontal 45° lines.

Chapter VII

PARALLEL PERSPECTIVE

XVIII. Figure 9 gives the projections of a right rectangular pyramid, so placed as to make a parallel perspective. It is the same size and placed at the same distances from the picture plane and horizon plane as in Fig. 2, in Part I. The pyramid has a base 23 feet below the horizon plane and is 7 feet by 12 feet. It has an altitude of 10 feet and the vertex is 24 feet behind the picture plane. The station is 15 feet in front of the picture plane, and is to the left of the center of the object 16 feet. In this drawing, as well as all orthographic projections of objects in this part, the $X X$ axis will be the intersection of the picture plane and the horizon plane. This makes the horizontal projection of the picture plane and the vertical projection of the horizon coincide with each other, and with the $X X$ axis. In the first article the horizontal plane was parallel to the horizon plane, while in this article it coincides with the horizon plane.

This figure also shows the horizontal projection of horizontal, 45° construction lines for the points *A*, *B*, *C*, and *D*. The horizontal projection for the 45° construction lines for *A* are $a^h a_2^h$ and $a^h a_3^h$. A third construction line for a check



Scale 1″ = 20 ft.

FIG. 9.

would be $a^h a_1^h$, perpendicular to the picture plane. In like manner we have for the point *B*, $b^h b_2^h$; $b^h b_3^h$, and $b^h b_1^h$.

In the same way for points *C* and *D*. The vanishing points of the 45° lines, the lines perpendicular to the picture plane, and the line *A B* are shown at $(V.\ P.)^h_l$; $(V.\ P.)^h_r$; $(V.\ P.)^h$, and $(V.\ P.)^1$, respectively.

In the case of the vanishing point of 45° lines we have formed an isosceles triangle, one leg being perpendicular to the picture plane, from the station, and intersecting it in the center of the picture. The other leg is drawn from the center of picture, and lying in the picture plane until it intersects the 45° horizontal lines through the station. Figure 9 shows one right-angle triangle in horiozntal projection as s^h, $(V.\ P.)^h$, $(V.\ P.)^h_l$, and the other as s^h, $(V.\ P.)^h$, $(V.\ P.)^h_r$. These two triangles are equal and hence $(V.\ P.)^h$ $(V.\ P.)^h_l$ equals $(V.\ P.)^h$ $(V.\ P.)^h_r$, and since these two triangles are isosceles, s^h $(V.\ P.)^h$ equals $(V.\ P.)^h$ $(V.\ P.)^h_l$ equals $(V.\ P.)^h$ $(V.\ P.)^h_r$, and hence the vanishing points of 45° horizontal lines are as far to the left or right of the center of the picture as the station is in front of the picture plane.

To draw the perspective *a, b, c, d* in Fig. 10 of the object *A B C D* by the use of Fig. 9 will now be described. First draw the horizon and 23 feet below it draw the line representing the intersection of the plane of the base of the pyramid with the picture plane. These two lines give the extreme vertical limits of the drawing. The point *B* is the farthest visible point of the object behind the picture plane as shown in Fig. 9, and the 45° lines from this point will intersect the picture plane farther to the left and right than any other 45° lines from a visible point of the object. *B* is 30 feet back of the picture plane, and hence where $b^h b^h_2$ intersects the picture plane is 30 feet to the left of *B*, and $b^h b^h_3$ intersects 30 feet to the right of *B*. The greatest width of the picture is now found to be 60 feet, while the greatest height is 23 feet. With these our perspective can be located in a proper position on our sheet.

On the base line, which is the intersection of the base plane of the object with the picture plane, in Fig. 10, locate b^v_2 and b^v_3 60 feet apart. The vertical projection of *B* is b^v 30 feet to the right of b^v_2. The station is 16 feet to the left of *A* and hence 12.5 feet to the left of *B*, which makes its projection s^v_1 in the base line 12.5 feet to the left of b^v or 17.5 feet to the right of b^v_2. The vertical projection of the station is in the horizon and directly above s^v_1, as shown at s^v, and is also the vanishing point of perpendicular horizontal lines, and is lettered V. P. The vanishing points of the 45° lines are 15 feet to the left and right of s^v, and are shown as $(\text{V. P.})_l$ and $(\text{V. P.})_r$. The point s^v_1 is the initial point for measuring all distances in the base line. The perpendicular and 45° lines through B intersect the base line in b^v, b^v_2, and b^v_3, which are 12.5 feet to the right, 17.5 feet to the left, and 42.5 feet to the right of s^v_1. The indefinite perspectives of these lines are b^v V. P.; b^v_2 (V. P.)$_r$; and b^v_3 (V. P.)$_l$, respectively, and they all meet in the point b, which is the perspective of B. The three lines through the point C intersect the base line in b^v, c^v_2, c^v_3, which are 12.5 feet to the right, 5.5 feet to the left, and 30.5 feet to the right of s^v_1. The indefinite perspective of these lines intersect in c the perspective of C. The three lines through D intersect the base line in d^v, d^v_2, d^v_3, which are 19.5, 1.5, 37.5 feet to the right of s^v_1. Their indefinite perspectives intersect in d, which is the perspective of D.

The horizontal plane passing through the vertex intersects the picture plane 10 feet above the base line or 13 feet below the horizon. The construction lines through A lie in this plane and intersect the picture plane. Two of these lines are sufficient to get the perspective of A, the third line serving as a check. These lines intersect the picture plane in the points a^v, a^v_2, a^v_3, which are 16 feet to the right, 8 feet to the left, and 40 feet to the right of s^v_1 and 10 feet above it. These indefinite perspectives intersect in the point a which is in the perspective of A.

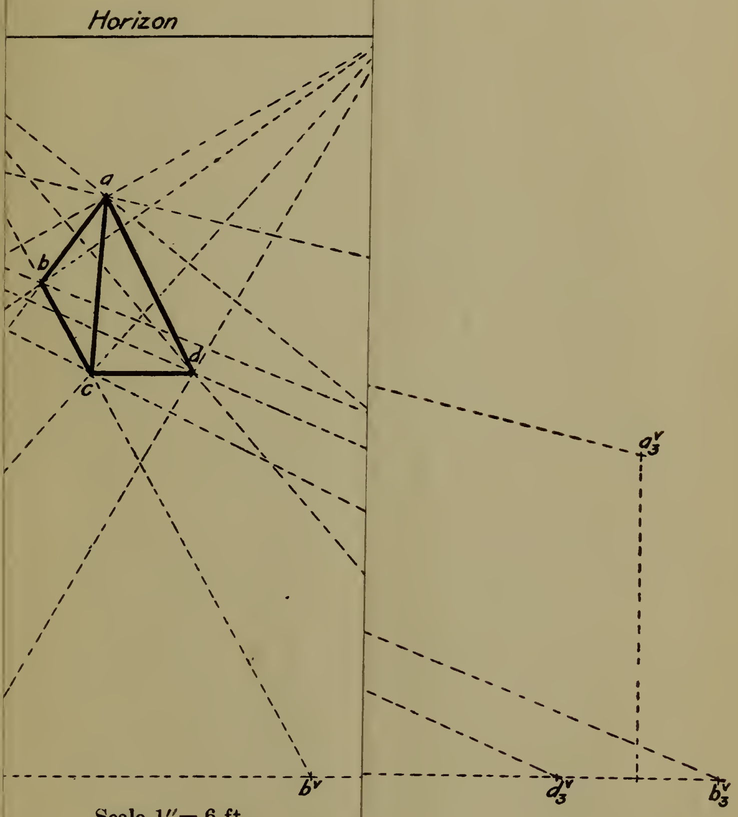
The perspective of the pyramid is now found by joining the points a, b, c, d in proper sequence, and we have the parallel perspective of the right rectangular pyramid $A\,B\,C\,D$. As a check $c\,d$ should be parallel to the horizon line, since $C\,D$ is a horizontal line and parallel to the picture plane. A check on $a\,b$ is to find the vanishing point of the line $A\,B$ as described under Fig. 2. In Fig. 10 we have computed the proper distances to the left of the vertical projection s^v of the station and below the horizon in order to scale and locate this vanishing point of $A\,B$ as shown in $(\text{V. P.})^1$. It is 8.75 feet to the left of s^v, and 25 feet below the horizon, and the line $(\text{V. P.})^1\,b$ produced should contain a as a check.

XIX. Figure 11 shows the projection of a building in which one end is parallel to the picture plane. The dimensions of the building are clearly shown in its projections, but the location of the building and the station with reference to the picture plane and the horizon plane are not shown in the drawings and will be given here as follows: near end is 16 feet behind the picture plane, the base of the building is 24.5 feet below the horizon plane, the station is in front of the picture plane 111 feet, and to the right of the center of the near end of the building 190 feet.

Figure 12 is the prespective of Fig. 11 by the method of scale, and is a more complicated case of parallel perspective than in Fig. 10. It is the same building and in the same location and position as in Fig. 3, Part I. You are thus able to compare the two methods and judge for yourself which method you consider the easier and quicker.

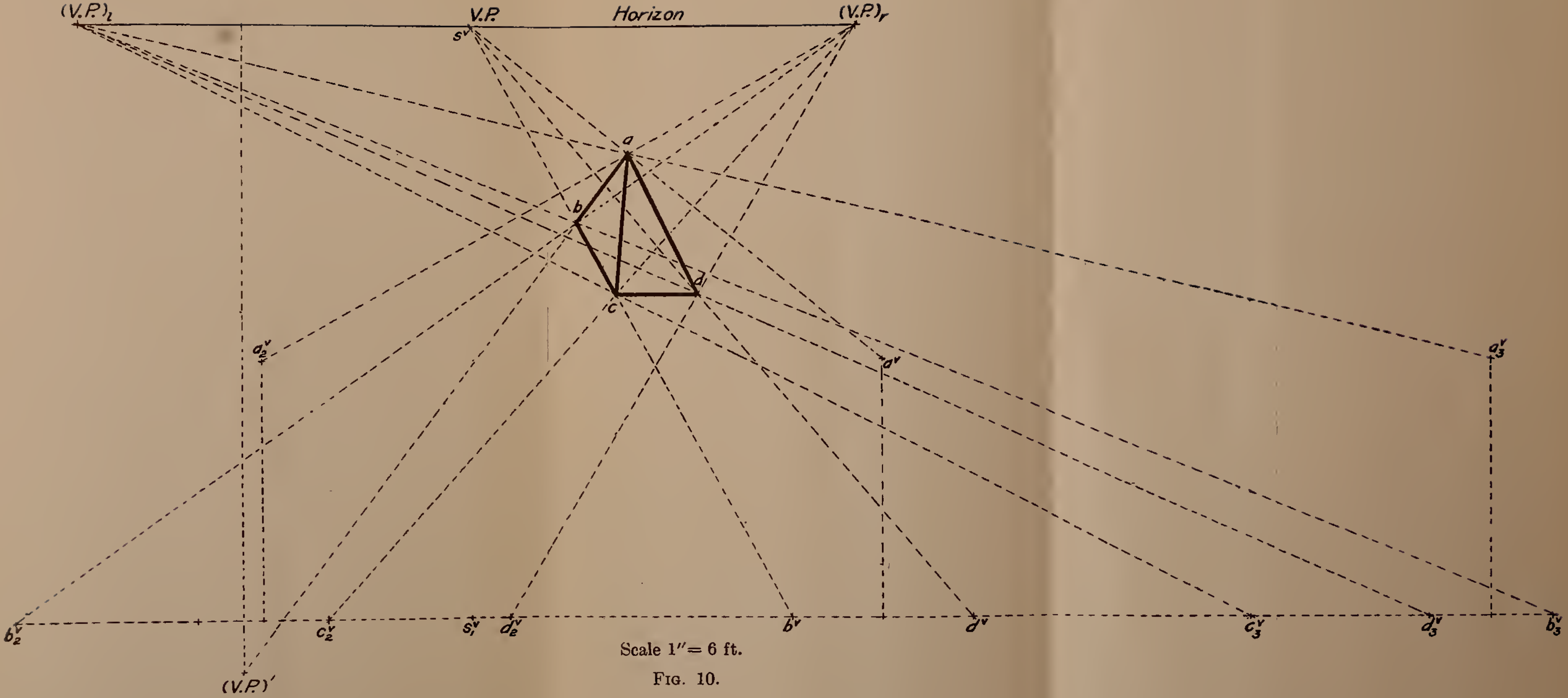
Let the horizon first be drawn, and 24.5 feet below it the line representing the intersection of the plane of the base and the picture plane. On this last line is where all 45° and perpendicular lines of the base intersect the picture plane.

On the horizon locate the center of picture or vertical projections s^v of the station, which is also the vanishing point V. P. for perpendicular lines. The vanishing points of the 45° lines are on the horizon, since they are horizontal lines, and as far to the left and right of the vertical projection of the station s^v as the station is in front of the picture plane, as $(\text{V. P.})_l$ and $(\text{V. P.})_r$, respectively. Now lay off on the horizon 190 feet to the left, and we have the point m^v. If the end view of the building were projected upon the picture plane, we would have the following outline $a^v\ o^v\ p^v\ r^v\ c^v\ c_1^v\ a_1^v$; $a_1^v\ c_1^v$ being 24.5 feet below m^v, and is in the intersection of the plane of the base of the building and the picture plane. The lines $a^v\ a_1^v$ and $c^v\ c_1^v$ are each perpendicular to



Scale 1″ = 6 ft.

FIG. 10.



Scale 1″ = 6 ft.

Fig. 10.

$a^v_1\, c^v_1$, and equal distances from m^v being one half the width of the building. The indefinite perspectives of the lines from the points a^v, a^v_1, c^v_1, c^v, etc., in the picture plane to infinity are drawn from a^v to V. P.; a^v_1 to V. P., etc., as described in the first part under Fig. 3. The definite perspective of the lines a^v V. P.; a^v_1 V. P., etc., are now found by the indefinite perspectives of the 45° lines through the points A, A_1, etc. The station projected from s^v onto the base line is shown at s^v_1, and this is the place from which all measurements are taken along the base line. The 45° lines through A_1 intersect the picture plane 221.5 and 189.5 feet to the left of s^v_1, and are shown as a^v_2 and a^v_3 respectively. They should intersect each other on the indefinite perspective a^v_1 V. P. as at a_1 which is the perspective of A_1. The 45° lines through C_1 intersect the picture plane 190.5 and 158.5 feet to the left of s^v_1, and are shown as c^v_2 and c^v_3 respectively. These lines vanish in $(\text{V. P.})_r$ and $(\text{V. P.})_l$ respectively. They should intersect each other in the indefinite perspective c^v_1 V. P. at c_1, which is the perspective of C_1. The perspectives of the points A and C are shown at a and c, and are obtained in the same way as a_1 and c_1, except in a plane parallel to the plane of the base and 30.5 feet above it. The perspective of P is found by finding the perspective of three lines passing through *P*, one being perpendicular and the other two making 45° with the picture plane. The perspective of the point where they meet is the perspective p of the point P. The perpendicular line and the 45° lines intersect the picture planes 190, 212, 168 feet to the left of s_1^v respectively, and 47 feet above it as shown at p^v, p_2^v, p_3^v. These three lines have their vanishing points in V. P.; (V. P.)$_r$; (V. P.)$_l$, and where they meet is the perspective p of the point P. The perspective of P_1 is on the indefinite perspective of p^v V. P. and in the plane of the near end of the building. One 45° line through P_1 intersects the picture plane 174 feet to the left of s_1^v and 47 feet above it as at p_1^v. The vanishing point of this line is (V. P.)$_l$ and its indefinite perspective is p_1^v (V. P.)$_l$ intersecting p^v V. P. in p_1 the perspective of P_1. The perspective of the other 45° line is not shown and would serve only as a check in the perspective p_1. Join $a\ p_1$ and $c\ p_1$ and we have the perspectives of $A\ P_1$ and $C\ P_1$. On these perspectives are the perspectives of O and R respectively, and hence the perspective of $O\ R$. $O\ R$ is a horizontal line and parallel to the picture plane, hence its perspective is parallel to the horizon. This line is 6 feet below P_1 therefore its vertical projection $o^v\ r^v$ will be 6 feet below the vertical projection

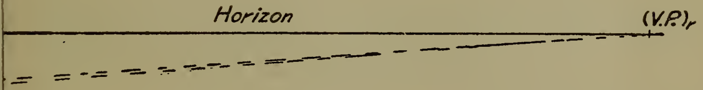
p^v of the point P. The middle point of $o^v r^v$ is b^v, which is the projection of B. The perspective of B is on a perpendicular through p_1 and where the indefinite perspective b^v V. P. intersects it is the perspective b of the point B. The perspective of $O R$ passes through b and is parallel to the horizon, and intersects $a\ p_1$ and $c\ p_1$ in o and r respectively. As a check the indefinite perspective o^v V. P. and r^v V. P. should pass through o and r respectively. We now have $a\,o$, $o\,r$, and $c\,r$ as the perspectives of $A\,O$, $O\,R$, and $C\,R$. The perspectives of $O\,P$ and $R\,P$ are now drawn and are shown as $o\,p$ and $r\,p$. As a check on these last two lines we can compute the distances of their vanishing points and locate them by scale. The vanishing point of $R\,P$ will be 111 feet above (V. P.)$_l$ and that of $O\,P$, 111 feet above (V. P.)$_r$. By connecting the points in proper sequence we have the perspective $a_1\,a\,o\,p\,r\,e\,c_1$ of the end $A_1\,A\,O\,P\,R\,C\,C_1$.

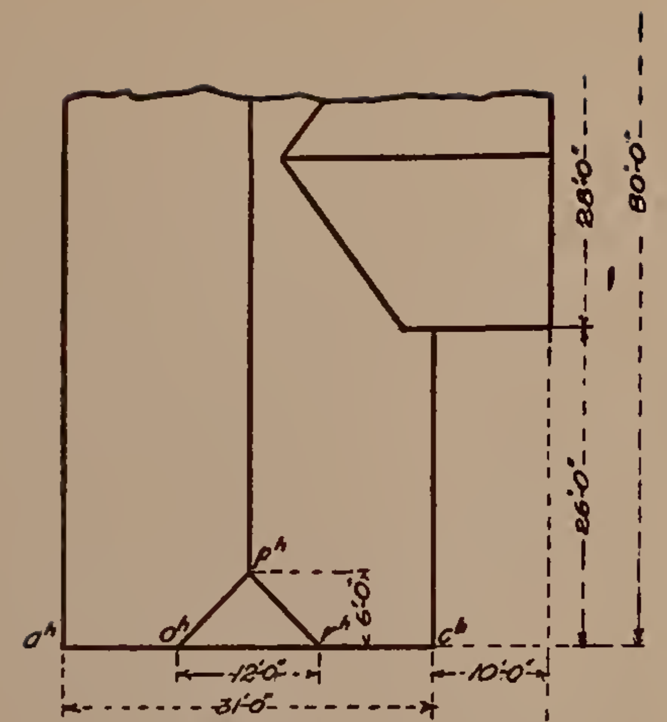
By the same method the perspective of F_1 can be found as shown at f_1. The construction lines intersect the picture plane 164.5; 206.5 (not shown) and 122.5 feet to the left of s^v_1. The first intersection is shown at f^v_2 and the third at f^v_3, and the second intersection would be used only as a check. By following the same methods you can locate the perspectives of all the corners of the building, and we have finally the perspective $a_1 a o p d f l k j$, etc. of the object $A_1 A O P D F L K J$, etc.

The above description with the aid of the additional construction lines should be enough to give a thorough understanding of the perspective in Fig. 12 and of the object shown in Fig. 11.

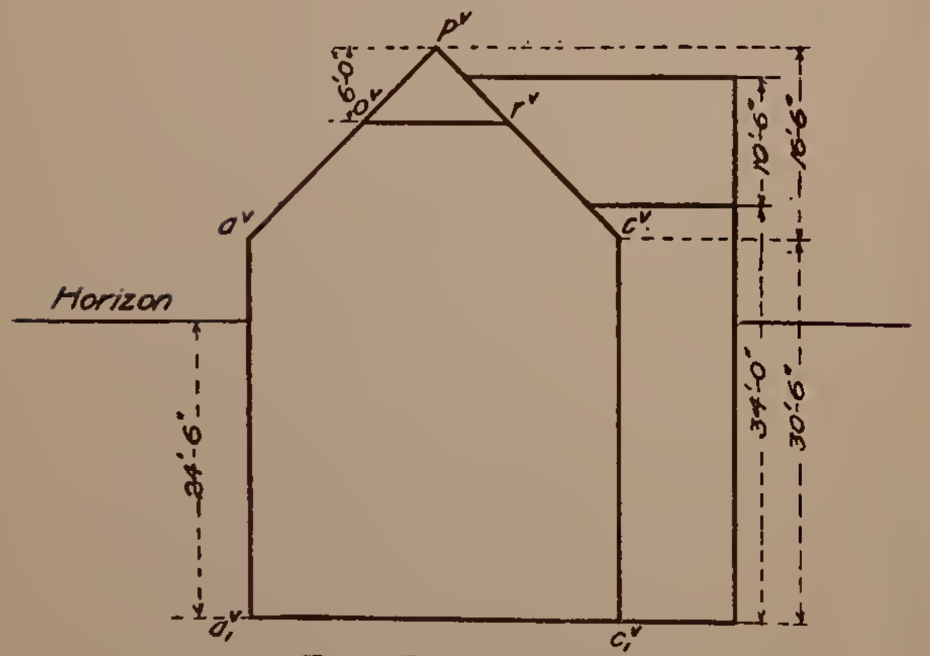
PROBLEM IN PARALLEL PERSPECTIVE

XX. Find the perspective of all the problems under parallel perspective in Part I, but solve them all by the method of scale.


Horizon
$(V.P.)_r$



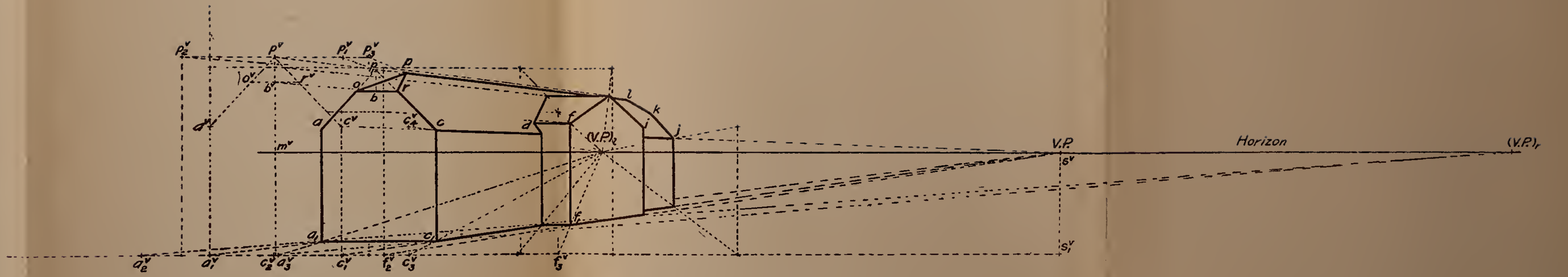
HALF PLAN



END ELEVATION

Scale 1″= 24 ft.

Fig. 11.



Scale 1″= 30 ft.

Fig. 12.

CHAPTER VIII

ANGULAR PERSPECTIVE

XXI. FIGURE 13 shows the working drawings of a cube and a parallelopiped block resting upon one edge of the cube. It occupies the same position as in Fig. 4. The faces of the cube being 30° and 60° with the picture plane, and the near edge of the cube 4 feet behind the picture plane. The base of the cube is 11.6 feet below the horizon plane. The station is 4.4 feet to the right of the near corner of the cube and 21.2 feet in front of the picture plane. This makes the vanishing point of 45° lines 21.2 feet to the left and right of the center of picture. Figure 13 also shows such computed distances as will enable you to find the distance to the left or right of the center of picture, and the distance above the base plane of the points where the perpendicular and 45° lines intersect the picture plane.

Figure 14 is the perspective of the object as shown in Fig. 13. The horizon is drawn, and 11.6 feet below it is shown the intersection of the plane of the base of the cube and the picture plane. The vertical projection of the station is shown at s^v, and is also the vanishing point of lines perpendicular to the picture plane, as well as the center of picture. The projection of this point on the base line is shown at s^v_1, from which measurements are taken to the left or right.

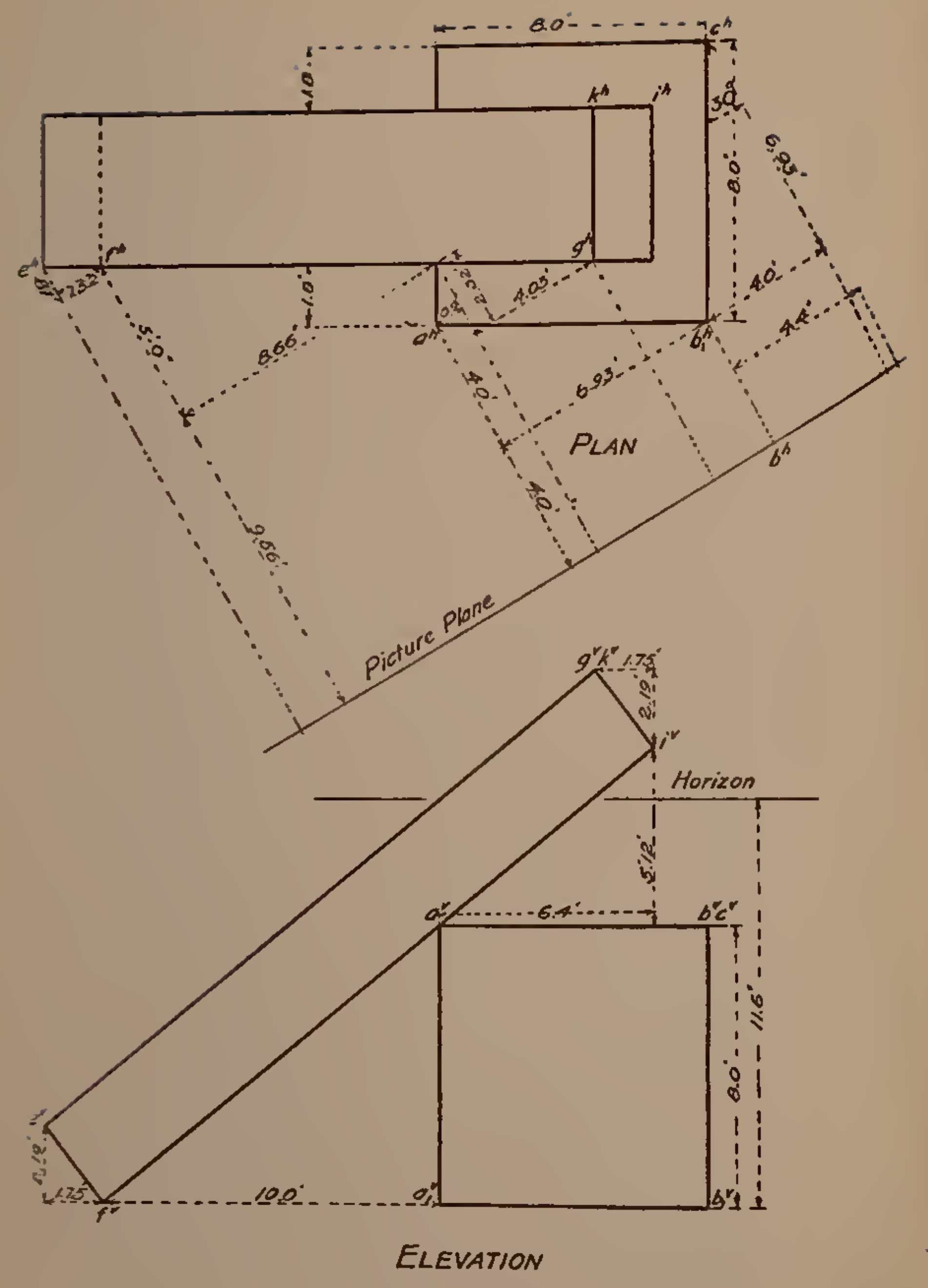
The perspective of the point B_1 will first be located. A perpendicular line through B_1 intersects the picture 4.4 feet to the left of the station, and its vertical projection is 4.4 feet to the left of s^v_1 as shown in b^v_1. The indefinite perspective of this perpendicular line is b^v_1 V. P. The two 45° lines intersect the base line in b^v_2 and b^v_3 ($4.4 \pm 4.0 = 8.4$ and 0.4), 8.4 and 0.4 feet respectively to the left of s^v_1. The indefinite perspectives of these 45° lines are b^v_2 (V. P.)$_r$ and b^v_3 (V. P.)$_l$ These three perspectives should meet in a point, and where they meet is b_1 the perspective of B_1. The perspective of A_1 is found by the same class of lines, and they intersect the picture plane in a^v_1, a^v_2, and a^v_3, each being 11.33, 19.33, 3.33 to the left of s^v_1. The perspective of C is in a plane 8 feet higher than the base plane. In this plane we use perpendicular and 45° lines as in the base plane, and they intersect the picture plane in c^v_{11}, c^v_2, c^v_3, which are 0.4, 11.33 feet to the left and 10.93 feet to the right of s^v_1 and 8 feet above the base plane. These three indefinite perspectives c^v_{11} V. P.; c^v_2 (V. P.)$_r$; c^v_3 (V. P.)$_l$ meet in the same point *c*, which is the perspective of *C*. In like manner the perspective of all the corners of the cube can be found and connected in the proper sequence, thus completing the perspective of the cube $A_1\ B_1\ B\ C\ D$. Another check is to find the vanishing points of the systems of lines $A\ B$, etc., and $B\ C$ etc. They can be computed and are in the horizon to the left and right of s^v 36.72, 12.24 feet, and are shown as (V. P.)^2; (V. P.)^1.

The perspective of the parallelopiped block will now be drawn. One edge of this block rests in the base plane, and we will find the perspective of the point *F* of this edge. The three construction lines that we wish to use intersect the picture plane on the base line in the points f^v, f^v_2, f^v_3, and are 19.09, 33.65, 4.53 feet to the left of s^v_1. The indefinite perspectives from these points meet in the point *f*, which is the perspective of *F*. The perspective of *E* can be found as you did *F*, but where the construction lines intersect the picture plane is in a line parallel to the base line and 2.19 feet above it. These points are shown at e^v_{11} e^v_1 e^v_2, and are 20.60, 36.03, 5.17 to the left of s^v_{111}. The perspective of *G* is *g* in a plane parallel to the base plane and 15.31 feet above it. The construction lines intersect the picture plane in

g^v, g^v_2, g^v_3; 6.41, 13.64 feet to the left and 0.82 feet to the right respectively of s^v_{11}. The other corners can be gotten in the same way and by connecting in proper sequence we have the perspective *e f g k i*, etc., of the parallelopiped block *E F G K L*, etc. Another check on this is to find the vanishing points of the systems of lines *E G*, etc. and *E F*, etc. These can be computed, and they lie directly below and above $(V.\ P.)^2$ respectively. Another check is that the block rests in the center of the edge *A D* of the cube.

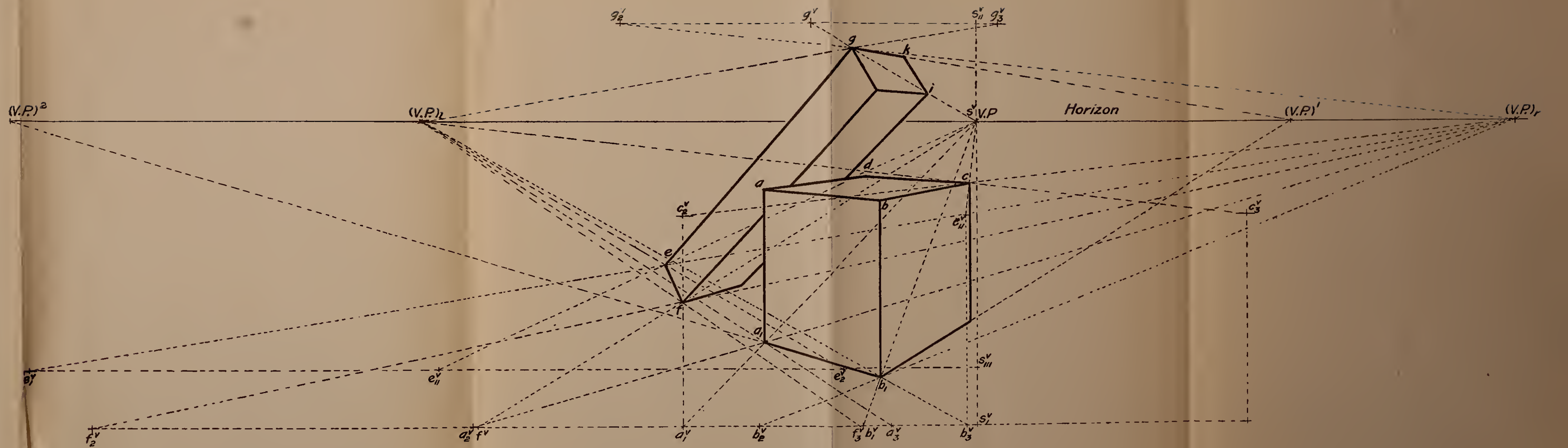
PROBLEMS IN ANGULAR PERSPECTIVE

XXII. Find the perspective of all the problems under angular perspective in Part I, but solve them all by the method of scale.



Scale 1″ = 8 ft.

Fig. 13.



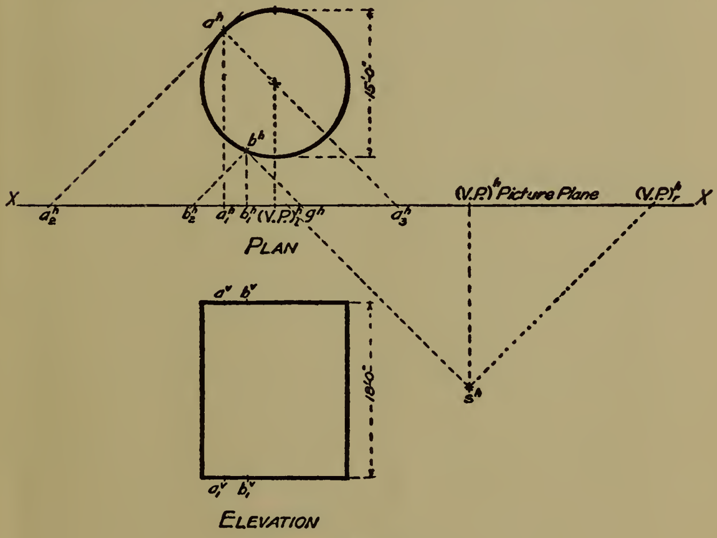
Scale 1″ = 4 ft.

Fig. 14.

CHAPTER IX

PERSPECTIVES OF CURVES

XXIII. FIGURE 15 shows the projections of a cylinder with its proper location from the picture and horizon planes. The cylinder is 15 feet in diameter and 18 feet long. The upper base is 10 feet below the horizon plane. The axis is parallel



Scale 1″ = 20 ft.

FIG. 15.

to the picture plane and 12.5 feet behind it. The station is to the right of the axis of the cylinder 20 feet and 18.75 feet in front of the picture plane. This drawing also shows the projections of two points, A, B and A_1, B_1 on each base of the cylinder, and the horizontal projections of the construction lines through these points. The points where these construction lines intersect the picture plane will have to be computed. The projections of the construction lines through A intersect the picture plane in a_1^h, a_2^h, a_3^h, through B in b_1^h, b_2^h, b_3^h, and the vanishing points in $(\text{V. P.})^h$; $(\text{V. P.})_r^h$; $(\text{V. P.})_l^h$.

Figure 16 shows the perspective of this cylinder in the position shown by Fig. 15. The horizon is first drawn and the vertical projection s^v of the station located. This is also the vanishing point V. P. The vanishing points $(\text{V. P.})_l$ and $(\text{V. P.})_r$ are also located being 18.75 feet to the left and right of s^v or V. P. The lower base line is located 28 feet below the horizon, and s_1^v is projected from s^v upon it, and is the initial point for measuring points in the lower base line. The construction lines through A_1 intersects the base line in a_1^v, a_2^v, a_3^v, and are 25.3, 43.1, 7.5 feet to the left of s_1^v. These indefinite perspectives meet in the point a_1 which is the perspective of the point A_1. The perspective of B_1 is b_1 at the intersection of the indefinite perspectives of the construction lines through B_1. They intersect the lower base line in b_1^v, b_2^v, b_3^v, at 22.87, 28.44, 17.30 feet to the left of s_1^v. The construction lines for the perspectives A and B intersect the picture plane at the same distances to the left of s_{11}^v, which is directly above s_1^v, as those for A_1 and B_1 intersect to the left of s_1^v, but in a plane 18 feet above the lower base plane, as shown at a^v, a_4^v, a_5^v, b^v, b_4^v, b_5^v. As a second check a is directly above a_1 and b directly above b_1, which are two elements of the cylinder. The perspective of the points in the upper base as well as in the lower base are connected by a smooth curve, and we have the perspectives of the upper and the lower bases. The extreme elements are drawn tangent to these perspectives of the bases, and this completes the perspective $a\ b\ a_1\ b_1$ of the cylinder $A\ B\ A_1\ B_1$.

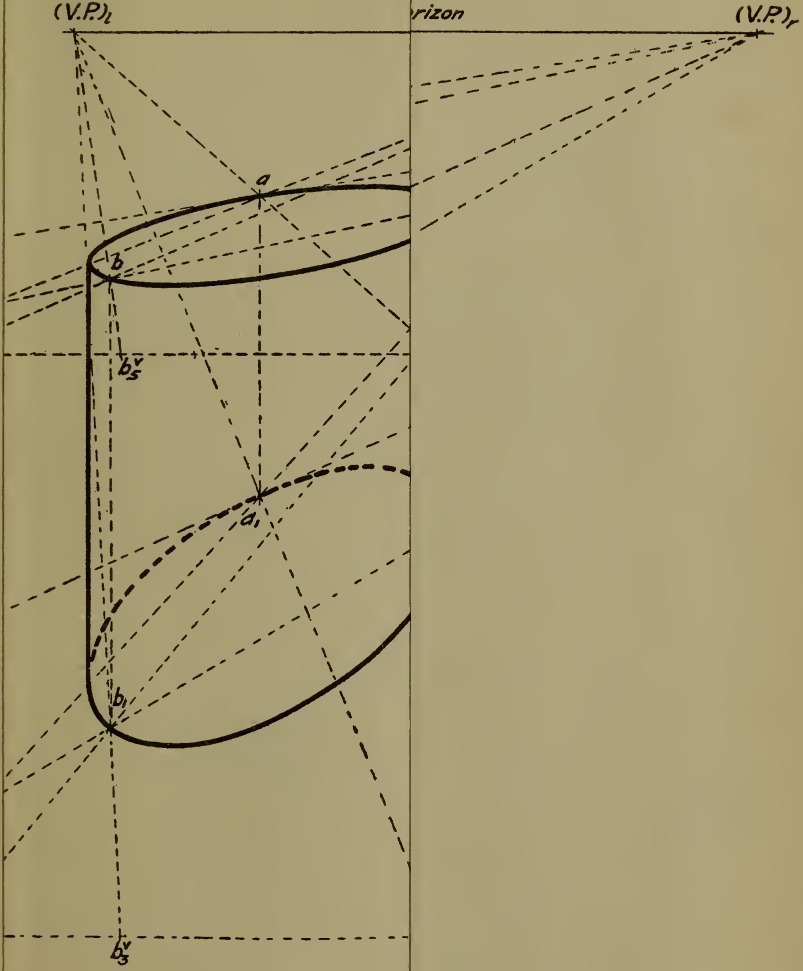
PROBLEMS IN PERSPECTIVE OF CURVES

XXIV. FIND the perspective of all the problems under perspective of curves in Part I, but solve them all by the method of scale.

Chapter X

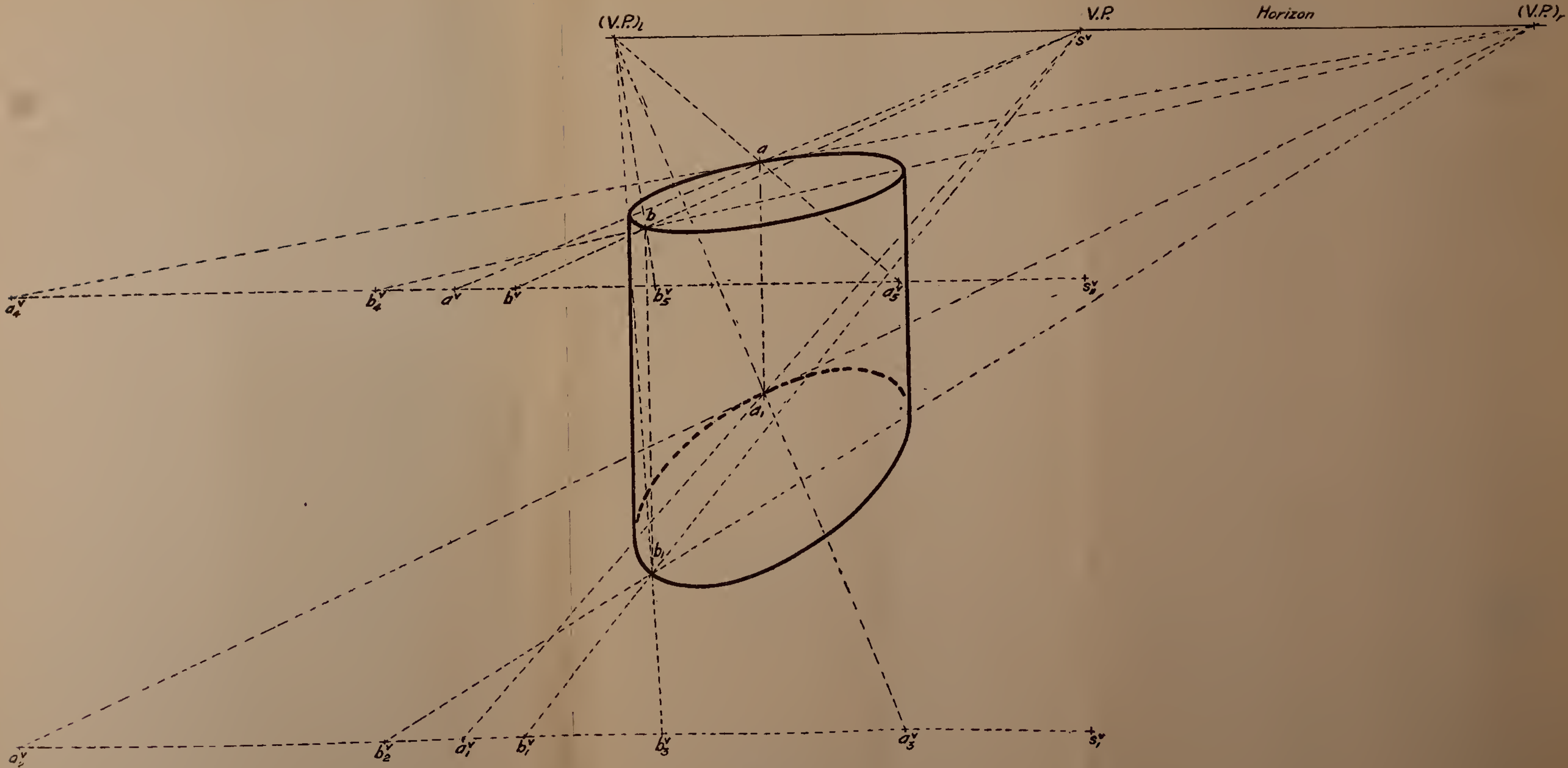
PERSPECTIVE OF SHADOWS

XXV. These are obtained by finding the perspective of the horizontal projection of the rays and the perspective of the rays themselves and where they intersect is the perspective of the shadow of the point. This is called the method by the vanishing point of rays. This method has been shown in Fig. 7, Part I, where the vanishing points were obtained by drafting methods. In using the method by scale the vanishing points are located by scaling the distance that the station is in front of the picture plane. The vanishing point of the horizontal projection of the rays is in the horizon, therefore their vanishing point $(V. P.)^3$, Fig. 7, is found by scaling on the horizon to the right of the vertical projection of the station s^v. The vanishing point of the rays themselves is in a line perpendicular to the horizon through $(V. P.)^3$, Fig. 7, and below it the distance that the station is in front of the picture plane. Having obtained the perspective of a point and its projection on the horizontal plane that the



Scale 1″ = 6 ft.

FIG. 16.



Scale 1″ = 6 ft.

Fig. 16.

shadow is on, we can now obtain the perspective of the shadow of this point as described under Fig. 7, Part I.

PROBLEMS IN THE PERSPECTIVE OF SHADOWS

XXVI. 1. Find the perspective of all the problems under perspective of shadows in Part I, but find your vanishing points by the method of scale.

2. Find the perspective of the shadow of the above objects on oblique planes, and on planes perpendicular to the horizontal plane, but oblique to the vertical plane.

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